

**EFFECTIVENESS OF HOUSING REVITALIZATION SUBSIDIES IN THE
PRESENCE OF ZONING**

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Abstract

This paper examines the impact of zoning on the effectiveness of uniform housing revitalization subsidies. Our analytical results show that, while zoning decreases the marginal return from housing improvements for zoned landlords, it also increases the equilibrium subsidy per housing improvement. This raises the possibility that an increase in zoning stringency can *increase* the effectiveness of housing revitalization subsidies by influencing the allocation of funds across space. Our simulation analysis presents a case where zoning increases subsidy effectiveness up to 26%. Additional numerical simulations illustrate how this result varies with the degree of zoning stringency, government budget, and the spatial distribution of housing quality.

Keywords: zoning, housing revitalization subsidy, targeting

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1. Introduction

Each year the US federal government provides approximately four billion dollars in housing revitalization subsidies for privately owned housing stock (Duda (2001)). These subsidies are usually allocated by the U.S. Housing and Urban Development department (HUD) to local governments as block grants. These subsidies aim to revitalize communities by stimulating private housing improvements and new construction, thereby internalizing some of the negative externalities associated with low-quality housing. Schwartz et al. (2006) find that the benefits of revitalization subsidies extend beyond those directly receiving them.¹ Despite substantial government spending, the resulting impact on housing and community quality remains weak in many cases (Urban Institute (1994), HUD (2003)).

Critics of these subsidies point to the way local governments allocate housing revitalization funds across space as a key reason for their ineffectiveness (Urban Institute (1994), HUD (2003)). However, restrictions on where revitalization funds can be spent, equity concerns, and politics can inhibit application of spatially-explicit targeting, and as such, uniform subsidies for all eligible neighborhoods remain the norm.²

An additional concern is that zoning may compromise the effectiveness of housing revitalization subsidies because of its distortionary impacts on new construction and on

¹ They estimate an aggregate increase in property values of about \$6.8 billion in a 2000-foot ring around the New York City Ten Year Plan housing site. Ellen et al. (2002) find similar spillover benefits from investments in owner-occupied housing.

² Most community development funding has regulatory constraints that determine the criteria that can be used to target funds. For example, Community Development Block Grant (CDBG) dollars restrict most funding to low and moderate-income households. Therefore, this major public revenue source for housing is only useful for housing revitalization when the target areas contain these high-need households. Spatially-explicit targeting funds implies that considerable resources shift between neighborhoods, which raises equity concerns and makes spatially-explicit targeting strategies very unpopular especially when resources shift from “high-need” to “lower-need” areas. In addition, if spatial demarcation coincides with racial or ethnic identity, the politics of spatial targeting can be explosive.

the returns from housing improvements (HUD (2005)). Empirical studies on the impacts of zoning on new construction show that zoning reduces the elasticity of housing supply to changes in housing prices (Gyourko and Saiz (2003), Glaeser and Gyourko (2005), Mayer and Sommerville (2000)). This in turn implies that the effectiveness of housing revitalization subsidies may be affected by the level of zoning stringency. To our knowledge, formal analyses of both the spatial effects of zoning when the quality and quantity of the housing stock are interrelated and the interaction of zoning with other urban policies have not been addressed in the zoning literature.

Using a consistent analytical and numerical general equilibrium framework, we examine how zoning affects total housing improvements. In other words, is zoning a barrier to the effectiveness of housing revitalization subsidies? In addition, we also aim to examine how zoning affects total housing supply when the market for housing quality is interrelated with the market for housing quantity. Changes in housing quality are capitalized into housing rental prices, which in turn can affect the amount of housing produced. These feedback effects are often overlooked in previous studies on housing market responses to government housing programs. There is also empirical evidence that the spillover effects of subsidized housing investment can be significant.³

Our analytical results show that zoning generates two opposing effects. On the one hand, zoning reduces housing improvements by restricting housing supply and thus the willingness of landlords to make housing improvements. We denote this the *zoning-*

³ Ioannides and Zabel (2003) provide strong evidence that neighbors' decisions to maintain, repair, renovate or make additions to their home induce individuals to increase their own repairs, maintenance and renovations. In particular, the authors estimate that the elasticity of housing maintenance with respect to the mean of the neighbors' housing maintenance is 0.660 and it is highly significant. This result is also consistent with Ioannides (2002) of an effect of maintenance by one's neighbors on own maintenance of 0.487.

interaction effect. On the other hand, because housing densities are not homogenous across space, zoning does not bind in all areas of the city. As a result, by reducing housing improvements in these zoned areas, additional funds are allocated to other neighborhoods who respond by increasing improvements. We denote this the *targeting effect*. This raises the possibility that an increase in zoning stringency may increase the effectiveness of revitalization subsidies by influencing the allocation of funds across space.⁴

Under plausible parameter values, our simulation results present a case where zoning is not a barrier to the effectiveness of housing revitalization subsidies, and in fact increases total housing improvements up to 26%. However, we also show that the effectiveness of revitalization funds is contingent upon the conditions of the subsidized neighborhoods and initial level of zoning stringency. For example, we show that the positive *targeting effect* dominates the negative *zoning-interaction effect* only for low-to-moderate levels of zoning stringency. Additional simulations with alternative spatial locations of housing quality demonstrate a case where zoning neither hinders nor helps subsidy effectiveness, and a case where zoning is a significant barrier to the effectiveness of housing revitalization subsidies.

The rest of the paper is organized as follows. In section 2 we present the analytical model and lay out the channels through which zoning influences the effectiveness of housing revitalization subsidies to increase total housing improvements and total housing

⁴ The spatial allocation of revitalization funds has been shown to be very important. For example, the spatially targeted Richmond Neighborhoods-in Bloom Program has yielded substantial benefits both in the targeted areas and surrounding neighborhoods, leading to more noticeable revitalization of the city's housing stock than when funds are uniformly available (Accordino et al. (2005), Rossi-Hansberg (2008)). Accordino et al. (2005) found that the average home sales price increased 9.9 percent per year faster in the NiB target neighborhoods than they did elsewhere in the city. Spillover effects in adjacent neighborhoods generated an increase in housing prices at a rate of 5.3 percent faster than the citywide average.

supply. In section 3 we describe the numerical model and in section 4 we present the simulation results. Finally, a brief conclusion follows in section 5.

2. Analytical Model

This section provides an analytical framework that offers insight into how zoning affects the allocation of housing improvement subsidies and thus, their effectiveness at increasing total housing improvements and total housing supply.

2.1. Model Assumptions

Consider an open city model, with all employment located at the central business district (CBD). Households are renters and identical. Absentee landlords own all the land and both land and housing markets are competitive. We assume that the opportunity cost of land in residential use is constant over time and equal to zero. As our analysis below uses a static framework, all the variables in our model reflect the market conditions of the current period.⁵

Housing bid rents

Households choose the amount of a numeraire composite good (Z) and consumption of housing (H), measured in square feet of floor space, in order to maximize their utility subject to their budget constraint, taking their building quality, $Q(x)$, and the aggregate quality of the city's housing stock, A , as given. Households' utility function is given by

⁵ While a static framework is sufficient for the results of this paper, an extension that examined the dynamic interaction of zoning with housing quality and quantity may produce additional insights. Durable housing models such as those developed in Wheaton (1982) and Arnott et. al. (1999) provide a useful framework for such an extension.

$U(u(H, Z), Q(x), A)$, where $U(\cdot)$ is continuous, quasi-concave and weakly separable in $u(\cdot)$, A and $Q(x)$. The household's budget constraint is given by $y = tx + pH + Z$ where y represents household income, t is transportation cost per mile, x denotes distance (in miles) from place of residence to the CBD and p is the rental housing price. The price of the composite good is set to one. Because of the open city assumption, in equilibrium, all households must receive the same level of utility \bar{V} . This condition implicitly determines households' housing bid rents, as the maximum rent per unit floor area that a household is willing to pay in the current period at a certain distance from the CBD if it is to receive \bar{V} :

$$p(y, t, Q(x), A, \bar{V}, x) \tag{2.1}$$

where $\frac{\partial p}{\partial Q(x)} > 0$, $\frac{\partial^2 p}{\partial Q(x)^2} < 0$, $\frac{\partial p}{\partial A} > 0$, $\frac{\partial^2 p}{\partial A^2} < 0$.

From (2.1), the greater the quality of the household's own building ($Q(x)$) and/or the aggregate quality of the city's housing stock (A), the higher the rent per unit floor area that a household is willing to pay at distance x from the CBD.

Housing supply and housing quality

Housing floor space is produced with land and capital according to a strictly concave, constant-returns production function. The intensive form of the production function is given by the concave function $h(S(x))$, where $S(x)$ is capital per unit of land (structural density) at location x . Construction costs $C^s(S(x))$ are a continuous convex function.

We assume there is a uniform upper limit on the square footage of housing (a floor-to-area ratio or FAR restriction) per unit of land.⁶ Thus,

$$h(S(x)) \leq \hat{h}(\hat{S}) \quad \forall x \quad (2.2)$$

where \hat{h} is the FAR limit per unit of land and \hat{S} is the total structural density associated with \hat{h} . We define zoning stringency as:

$$\hat{\lambda} = S_0 - \hat{S} \quad (2.3)$$

where S_0 is an exogenous “reference” structural density such that $S_0 > \hat{S}$.⁷ Note that an increase in stringency $\hat{\lambda}$ implies a decrease in \hat{S} and thus, in the FAR limit \hat{h} .

The quality of the housing stock, $Q(x)$, at location x is defined by:

$$Q(x) = \bar{Q} - \delta(x) + I(x) \quad (2.4)$$

where $\delta(x)$ is an exogenous level of physical deterioration of the housing stock at location x , and $I(x)$ is the level of housing improvements.⁸ \bar{Q} represents the “maximum” housing stock quality, with no deterioration or needed improvements. According to (2.4) the quality of the housing stock can be fully or partially offset by housing improvements: $0 \leq I(x) \leq \delta(x)$. Total improvements are thus given by:

$$\bar{I} = \int_0^{\bar{x}} I(x) 2\pi x dx \quad (2.5)$$

⁶ For simplicity, we consider a uniform level of zoning throughout the city and that zoning stringency does not change over time. Nonuniform zoning would not alter our qualitative results, but would make the analytical analysis more cumbersome.

⁷ While this definition appears excessively complicated, it simplifies the following analytics and facilitates interpretation of the comparative statics derived in subsection 2.3.

⁸ Our analysis uses a static framework, and thus for simplicity we have not modeled the dynamic process that has led to the deterioration of the housing stock in the current period. Reasons for the presence of housing deterioration in a certain period include housing durability (Glaeser and Gyourko (2005)), construction costs (Gyourko and Saiz (2003)), and externalities from individual behaviors (Kutty (1995)).

Housing improvement costs are given by $C'(S(x), I(x))$ which is continuous and convex. The aggregate quality of the city's housing stock in the current period is thus represented by:

$$A = \int_0^{\bar{x}} Q(x) 2\pi x dx = \int_0^{\bar{x}} (\bar{Q} - \delta(x)) 2\pi x dx + \bar{I} \quad (2.6)$$

where \bar{x} is the (endogenous) city boundary. Thus, improvements made at a given location, $I(x)$, increase the aggregate quality of the housing stock, A , which is in turn capitalized into housing prices across the city per equation (2.1).

Subsidized Housing Improvements

Because landlords do not recognize the citywide effects of their private improvements, landlords under-improve their housing stock, leading to suboptimal housing quality from a social perspective. Recognizing this externality, government provides a subsidy to induce higher housing improvements. The government budget is assumed to be balanced:

$$g \int_0^{\bar{x}} I(x) h(S(x)) 2\pi x dx = M \quad (2.7)$$

where total subsidy payments - defined as the per unit subsidy, g , times the square footage of housing improvements - equal an exogenous housing revitalization budget M . Note that the marginal cost to the government of a unit of housing improvement is $gh(S(x))$.

2.2. Optimal private housing improvements and optimal construction

Landlords problem

The return per acre of land in residential use at a particular location x is defined as:

$$R_n(x) = \max_{I(x), S(x)} p(y, t, Q(x), A, \bar{V}, x)h(S(x)) - C^s(S(x)) - C^I(S(x), I(x)) + gI(x)h(S(x)) \quad (2.8)$$

where $Q(x)$ is given by (2.5).

A landlord at location x chooses the level of structural density, $S(x)$, and the level of housing improvements, $I(x)$, in order to maximize (2.8) taking zoning stringency (2.3), the housing improvement subsidy g and the aggregate housing quality (2.6) as given. From this maximization problem we obtain the optimal structural density and the optimal level of improvements, respectively as:

$$S^*(y, t, \delta(x), \bar{I}, \bar{V}, \hat{\lambda}, g, x) \geq 0 \quad \text{and} \quad I^*(y, t, \delta(x), \bar{I}, \bar{V}, \hat{\lambda}, g, x) \geq 0 \quad (2.9)$$

Equation (2.9) highlights several important features of the model. First, because housing bid rents capitalize housing quality, structural density depends on both the (exogenous) physical deterioration of the local housing stock $\delta(x)$ and citywide total housing improvements \bar{I} . In addition, zoning stringency ($\hat{\lambda}$) also affects housing improvement decisions. Zoning reduces housing densities and thus, the marginal return of housing improvements in locations where it is binding. As a result, any policy affecting housing density affects housing quality and thus the choice of housing improvements. Second, feedbacks between landlords are captured as interdependencies between individual improvement decisions $I^*(x)$ and total improvements \bar{I} .⁹ Our model

⁹ The interdependence of improvement choices implies that they form a Cournot-Nash equilibrium in which each landlord chooses an improvement level that is the best response to the improvements chosen by his neighbors. To ensure stability of the equilibrium we assume $-1 \leq \frac{\partial I(x)}{\partial \bar{I}} \leq 1$. The interaction between

accommodates both strategic complementarities when $\partial I^*(x)/\partial \bar{I} > 0$, and congestion/repelling effects when $\partial I^*(x)/\partial \bar{I} < 0$. In addition, spatial heterogeneity in housing deterioration and housing density implies that landlords respond differently to the same level of housing subsidy g .

Closing conditions

The equilibrium city boundary, \bar{x} , is implicitly yielded by:

$$R_n(y, t, Q(\bar{x}), \bar{I}, \bar{V}, \hat{\lambda}, g, \bar{x}) = 0 \quad (2.10)$$

Equation (2.10) requires that, in equilibrium, the residential land rent equals the opportunity cost of vacant land (0) at the city boundary (\bar{x}).

Given the city boundary defined by (2.10) and optimal improvements given by (2.9), equation (2.7) implicitly defines the housing revitalization subsidy per unit of housing improvement that balances the government budget constraint as:¹⁰

$$g = g(y, t, \bar{I}, \bar{V}, \hat{\lambda}, \delta(x), M) \quad (2.11)$$

From (2.9) and (2.11), equilibrium aggregate housing improvements, $\bar{I}(y, t, \bar{V}, \hat{\lambda}, g, M)$, is implicitly given by:

$$\bar{I} = \int_0^{\bar{x}} I^*(x) 2\pi x dx \quad (2.12)$$

Similarly, the equilibrium aggregate housing supply, $\bar{h}(y, t, \bar{I}^*, \bar{V}, \hat{\lambda}, g)$, is given by:

landlords in our analytical model is not differentiated as a function of distance. However, in our simulation model, we allow interaction effects to decay with distance.

¹⁰ Inserting the optimal level of housing density and housing improvements that maximize (2.9) for each location in the city as well as the equilibrium city boundary into (2.7) and solving for g , yields the unitary subsidy that balances the government budget constraint in equilibrium.

$$\bar{h} = \int_0^{\bar{x}} h(S^*(x))2\pi x dx \quad (2.13)$$

In the following subsection, comparative static analysis on the equilibrium equations above (2.11), (2.12), (2.13) is used to decompose the effect of zoning on the effectiveness of housing revitalization subsidies.

2.3. The impact of increases in zoning stringency on total improvements and total housing supply

We now consider the impacts of a marginal increase in zoning stringency, $\hat{\lambda}$, on private improvements ($I(x)$), subsidy per unit of housing improvement (g), total housing improvements (\bar{I}) and total housing supply (\bar{h}). Complete derivations are provided in Appendix A.

Impacts of increases in zoning stringency on individual improvements and subsidy per unit of housing improvement

The impact of a marginal increase in zoning stringency on individual improvements, *ceteris paribus* is given by:

$$\frac{\partial I(x)}{\partial \hat{\lambda}} = - \frac{\left(\frac{\partial p(x)}{\partial I(x)} + g \right) \frac{\partial h(x)}{\partial S(x)} \frac{\partial S(x)}{\partial \hat{\lambda}}}{\frac{\partial^2 (p(x)h(x))}{\partial I(x)^2} - \frac{\partial^2 C^I(x)}{\partial I(x)^2}} \quad (2.14)$$

Equation (2.14) captures the direct effect of the marginal increase in zoning stringency on individual housing improvements for a given level of housing subsidy and

level of aggregate housing improvements.¹¹ Increasing zoning stringency reduces the housing supply, thus reducing the return on housing improvements, which is captured in the numerator.¹² Thus, $\partial I(x)/\partial \hat{\lambda} \leq 0$ and any developer in locations where zoning becomes more binding will decrease their housing improvements. This reduction in housing improvements for zoned neighborhoods means that the government budget constraint is no longer balanced. For the budget to balance, subsidy g must increase:

$$\frac{dg}{d\hat{\lambda}} = \frac{-g \int_0^{\bar{x}} h(S(x)) \frac{\partial I(x)}{\partial \hat{\lambda}} 2\pi x dx}{g \int_0^{\bar{x}} h(S(x)) \frac{\partial I(x)}{\partial g} 2\pi x dx + \int_0^{\bar{x}} h(S(x)) I(x) 2\pi x dx} \quad (2.15)$$

Equation (2.15) represents the increase in the housing improvement subsidy following the increase in zoning stringency. The release of funds from areas where zoning reduces the return on improvements increases the subsidy offered to other developers. In this sense, zoning can serve as an indirect targeting mechanism by shifting the allocation of funds across space.

The numerator in (2.15) represents the amount of subsidies released from neighborhoods where zoning is binding. This equals the subsidy per unit of improvement per square foot multiplied by the decrease in total improvements. The denominator in (2.15) is the marginal subsidy payment, which is positive. Thus, $dg/d\hat{\lambda} \geq 0$. Because an

¹¹ Note that the total effect of zoning on individual improvements is given by:

$$\frac{dI(x)}{d\hat{\lambda}} = \underbrace{\frac{\partial I(x)}{\partial \hat{\lambda}}}_{\text{direct effect}} + \underbrace{\frac{\partial I(x)}{\partial g} \frac{dg}{d\hat{\lambda}} + \frac{\partial I(x)}{\partial \bar{I}} \frac{d\bar{I}}{d\hat{\lambda}}}_{\text{indirect feedback effects}}. \text{ The sign of this equation depends on the magnitude of}$$

the feedback effects and direct effect. In our simulations, we explore the magnitude of these effects.

¹² The denominator in (2.14) measures the curvature of the housing bid rent function, which is always negative due to concavity.

increased subsidy lowers the cost of housing improvements, there is an increase in individual improvements, $\partial I(x)/\partial g \geq 0$, in the rest of the city.

Impacts on total housing improvements

By totally differentiating (2.12) with respect to $\hat{\lambda}$, the total impact of a marginal increase in zoning stringency on total housing improvements can be expressed as:

$$\frac{d\bar{I}}{d\hat{\lambda}} = \left[\underbrace{\int_0^{\bar{x}} \frac{\partial I(x)}{\partial \hat{\lambda}} 2\pi x dx}_{d\bar{I}^Z} + \underbrace{\int_0^{\bar{x}} \frac{\partial I(x)}{\partial g} \frac{dg}{d\hat{\lambda}} 2\pi x dx}_{d\bar{I}^T} \right] * \left[\underbrace{1 - \int_0^{\bar{x}} \frac{\partial I(x)}{\partial \bar{I}} 2\pi x dx}_{d\bar{I}^M} \right]^{-1} \quad (2.16)$$

Equation (2.16) decomposes the impact of zoning into three key components. The term labeled $d\bar{I}^Z$ represents the reduction in improvements from the *zoning-interaction effect*. This is the direct loss in total improvements due to restrictions on additional structural density. The term labeled $d\bar{I}^T$ is the increase in improvements from the *targeting effect*. It represents the increase in total improvements as zoning increases the subsidy per unit of improvement. This effect equals the increase in improvements induced by the subsidy multiplied by the increase in the level of subsidy. Thus, $d\bar{I}^Z + d\bar{I}^T$ represents the net effect of a marginal increase in zoning stringency on total improvements. While the *targeting effect* is strictly positive, it is unclear if it can offset the losses from the *zoning-interaction effect*. In the simulation results in section 4, we explore the magnitude of these two effects for a representative city.

Finally, the third component, indicated by the last term in (2.16), $d\bar{I}^M$, is the *multiplier effect*. This term captures the effect of total improvements, \bar{I} on individual improvement decisions. This term is always positive; however its magnitude will depend

on the relationship between $I(x)$ and \bar{I} . If $I(x)$ and \bar{I} are substitutes, then $\partial I(x)/\partial \bar{I} < 0$ and $0 < d\bar{I}^M < 1$, but if $I(x)$ and \bar{I} are complements, then $\partial I(x)/\partial \bar{I} > 0$ and $d\bar{I}^M > 1$. The net effect on aggregate improvements from an increase in zoning stringency, $d\bar{I}^Z + d\bar{I}^T$, is thus amplified in the presence of complementarities across housing improvements.

Together equations (2.14), (2.15) and (2.16) suggest that zoning may not necessarily be a barrier to the effectiveness of housing improvements subsidies. While zoning reduces total effectiveness for some locations (*zoning-interaction effect*), the *targeting effect* may channel funds to locations where they will have the greatest impact per dollar invested.¹³ However, because of spatial heterogeneity in housing density and level of physical deterioration it is unclear from the analytical model if the “right” locations are being zoned and if the “right” locations receive the subsidy. In our numerical model, we explore how these features impact the *zoning-interaction effect*, *targeting effect*, and *multiplier effect*, and ultimately whether or not zoning can actually increase aggregate improvements and aggregate housing supply.

Impacts on housing supply

By totally differentiating aggregate housing supply (2.13) with respect to $\hat{\lambda}$ while taking into account (2.9) in the presence of a subsidy, the total impact of a marginal increase in zoning stringency on aggregate housing supply can be expressed as:

¹³ This result is similar to Nichols and Zeckhauser (1982) who look at the efficiency of welfare programs in the context of in-kind transfers. Though less efficient than a cash payment, the authors show that in-kind transfers serve as a mechanism to screen legally eligible agents, channeling funds towards the agents in most need.

$$\begin{aligned}
\frac{d\bar{h}}{d\hat{\lambda}} = & \int_0^{\bar{x}} \frac{\partial h(x)}{\partial S(x)} \left[\underbrace{\frac{\partial S(x)}{\partial \hat{\lambda}}}_I + \underbrace{\frac{\partial S(x)}{\partial g} \frac{dg}{d\hat{\lambda}}}_{II} \right] 2\pi x dx + \\
& + \underbrace{\int_0^{\bar{x}} \frac{\partial h(x)}{\partial S(x)} \frac{\partial S(x)}{\partial \bar{I}} \frac{d\bar{I}}{d\hat{\lambda}} 2\pi x dx + h(S(\bar{x})) 2\pi \bar{x} \frac{d\bar{x}}{d\hat{\lambda}}}_{III}
\end{aligned} \tag{2.17}$$

Term *I* in (2.17) represents the direct, strictly negative change in housing supply from the increased zoning stringency. This effect is consistent with empirical studies on the effects of zoning on housing supply that find that housing regulations reduce housing supply (Mayer and Sommerville (2000), Glaeser and Gyourko (2005)). Term *II* represents the change in housing supply due to the reallocation of housing improvements subsidies within the city.

Finally, the third term *III* captures the spillover effects of aggregate housing improvements being capitalized into housing bid rents and thus stimulating housing supply. The spatial spillover effects in term *III* are absent in, for example, Lin et. al. (2004) and Hoff and Sen (2005) because in these studies, housing quantity and housing quality are not jointly determined and housing submarkets are not interdependent through spatial externalities. However, term *III* is consistent with the empirical evidence that removing negative externalities due to housing deterioration increases property values, which in turn stimulates housing supply in neighboring areas (Schwartz et al. (2006), Rossi-Hansberg et al (2008)).

If zoning increases aggregate housing improvements, the change in housing supply, $d\bar{h} / d\hat{\lambda}$, will be positive if losses due to increased zoning stringency (term *I*) are less than the increase in housing supply due to the capitalization of increased improvements into housing prices (terms *II* and *III*).

3. Calibration of the Numerical Model

The formal structure of our numerical model is similar to the analytical model of section 2.1; however the numerical model incorporates more realistic features of the housing market, such as a complimentary relationship between private and aggregate improvements, a centrally located area of low quality housing, and spillover effects between housing markets that decrease with distance. In this section we focus on the calibration of the key parameters of our numerical model. It should be noted that the model is not calibrated for any specific metropolitan area. However, we have captured the general key features of metropolitan areas based on the existing literature. Table 1 summarizes our benchmark parameter values. Appendix B provides further discussion of the functional forms used in the numerical model.¹⁴

Valuation of Improvements: Based on empirical studies that examine the change in property values in response to a housing revitalization subsidy (Ellen et al. (2003), Schwartz et al. (2006)), we calibrate our valuation parameters to generate an average WTP for housing improvements equal to \$1.25 per dollar of subsidy spent. Furthermore, a complimentary relationship between private improvements and aggregate improvements is calibrated based on the results in Ioannideas (2002).¹⁵

Initial Level of Housing Improvements: We have calibrated our benchmark such that no landlords make housing improvements before the subsidy is provided. This assumption removes complications arising from crowding out of private investments

¹⁴ The simulation model was solved using *Mathematica*. Details of the computer programs and calibration are available from the authors upon request.

¹⁵ Our parameter value implies that, while households receive utility from living in a nice house in a city with low-quality housing stock or living in a bad house in a city with high-quality housing stock, they receive even more utility from living in a nice house in a city with high-quality housing stock.

when a subsidy is given. In our sensitivity analysis, we relax this assumption to allow for positive improvements pre-policy.

Spatial distribution of low quality housing: We assume 20% of the housing stock exhibits some level of physical deterioration. According to the 2001 American Housing Survey (AHS), national sample, 19% of the existing housing units exhibited deficiencies in terms of external conditions such as a sagging roof, sloping outside walls and broken windows. The benchmark model is calibrated such that these deteriorated units are located between 0 and 3.5 miles from the CBD, with more deteriorated units found near the CBD.

Spillover Effects: Our spillover effects were calibrated to mimic empirical studies by Ellen et al. (2002, 2003), which suggest that the impact of low quality housing on neighboring areas diminishes at a distance of 2000 feet, and disappears as one moves even further away.¹⁶ In addition, our functional form for spillover effects also captures a “flight from blight” effect due to differences in housing quality between the city and the suburbs as suggested in the empirical literature (Bayoh et al. (2006)).

Zoning Stringency: We calibrate zoning stringency to be consistent with empirical studies that show the growth in the housing stock due to new construction is considerably lower in cities with greater regulatory stringency (e.g. Glaeser and Gyourko (2005), Quigley and Raphael (2005)). For our benchmark, we have chosen our uniform zoning stringency to correspond to a 10% reduction in housing supply. Because the magnitudes of the effects described in the analytical section depend critically on the level of zoning,

¹⁶ For example, Ellen et al. (2002) investigated the neighborhood price impacts of the Nehemiah program and the Housing Partnership program in New York City during the 80’s and 90’s. They identified a positive home price impact for both programs, attenuating over distance: 11% within 500 feet, 6% at 1000 feet and 3% at 2000 feet.

we vary zoning stringency to analyze its impact on total improvements and total housing supply.

Housing revitalization budget: We calibrate the housing revitalization budget consistent with values of federal funding initiatives for the revitalization of blighted urban areas. For example, in 2002 the Community Development Block Grant program (CDBG) funding averaged \$2.9 million per eligible city or county and the HOPE IV funding averaged \$2.6 million per site. While federal funds account for around two-thirds of revitalization funds, they are typically augmented by local taxation and other funding sources. Thus, we have chosen an exogenous budget equal to \$4 million, and we perform sensitivity analysis of this parameter.

4. Simulation Results

This section presents our simulation results. Subsections 4.1 and 4.2 present the spatial distributions of housing improvements and housing supply in the presence and absence of zoning under our benchmark parameters. It should be noted our emphasis is on qualitative rather than quantitative differences in the effectiveness of housing revitalization subsidies and to decompose the effects of zoning as discussed in the analytical model. The quantitative differences can vary depending on the zoning stringency, the degree and location of low quality housing, and the amount of housing subsidies, which we explore in subsections 4.3 and 4.4. In particular, we are interested in whether zoning can increase total housing improvements and/or housing supply under reasonable parameter assumptions.

4.1. Improvements across Space

We first examine the level of improvements across the central city area (0 to 3.5 miles from the CBD) under a housing revitalization subsidy in the absence and presence of zoning. In figure 1, on the horizontal axis we measure distance in miles from the CBD. On the vertical axis, we measure improvements per unit of existing housing stock. The main goal of figure 1 is to decompose the impacts of zoning on total improvements as discussed in section 2 and to illustrate how zoning may influence the allocation of housing subsidies and thus, housing improvements across neighborhoods.

The solid curves $I^{NZ}(x)$ and $I^Z(x)$ show the equilibrium level of improvements where we set zoning stringency at zero and 17,500 sq. ft., respectively.¹⁷ Note that even in the absence of zoning, there will be uneven participation in the housing revitalization program across neighborhoods. This occurs because neighborhoods differ in their level of housing deterioration and therefore respond differently to the same level of subsidy. Thus, when we add zoning, the set of landlords who make improvements (and the magnitude of those improvements) changes as the level of subsidy per unit of housing improvement also changes.

To decompose the impacts of zoning on total improvements we add two additional curves to figure 1. $I_o^Z(x)$ displays improvements in the presence of zoning, holding the per-unit subsidy and multiplier effects constant. The shaded area between $I^{NZ}(x)$ and $I_o^Z(x)$ isolates the *zoning-interaction effect*. Because zoning reduces the return of housing improvements in binding neighborhoods, total improvements in neighborhoods

¹⁷ For a reference point, we use the amount of total housing supply at the CBD in the absence of zoning. Thus, under this zoning stringency, the landlord at the CBD is restricted to build 17,500 sq. ft. less than he would in the absence of zoning.

located between 0.95 and 1.55 miles from the CBD are smaller. Under our central estimates, the *zoning-interaction effect* represents around a 59% decrease in total improvements relative to the absence of zoning. However, because the government budget constraint is not yet balanced, this curve does not represent equilibrium. The direction of this partial equilibrium effect is consistent with empirical findings by Tanzer (1985) who finds that a decrease in property tax increased housing quality. Here, we have an increase in zoning (comparable to an increase in property tax) leading to a decrease in housing quality. Zoning affects housing improvements because landlords' decisions on additions to structural densities and housing improvements are interrelated.

$I_{oo}^Z(x)$ shows improvements in the presence of zoning when the subsidy per unit of improvement adjusts to balance the government budget constraint, while still holding multiplier constant. The shaded area between $I_o^Z(x)$ and $I_{oo}^Z(x)$ isolates the *targeting effect*. This effect causes an upward shift of the improvements curve $I_o^Z(x)$. By reducing improvements in some neighborhoods, zoning increases the revitalization subsidy and more revitalization funds are transferred towards the remaining neighborhoods (those located between 1.4 and 2.3 miles from the CBD). Under our central parameter values, the *targeting effect* represents a 78% increase in total improvements relative to the absence of zoning.

Finally, the shaded area between $I_{oo}^Z(x)$ and $I^Z(x)$ captures the final adjustment to the equilibrium level of improvements due to the *multiplier effect*. Complementarity between individual improvements and aggregate improvements implies that the *multiplier effect* is greater than 1. Because the net impact of the *zoning-interaction* and *targeting effects* is positive, total improvements increase more in the presence of zoning.

The *multiplier effect* results in positive feedbacks, which generate additional improvements for neighborhoods located between 1.65 and 2.35 miles. Under our central parameter values, this effect represents a 7% increase in total improvements relative to the absence of zoning. Note that the equilibrium level of improvements would be underestimated and the *multiplier effect* would be completely absent in housing market models (such as Arnott et. al. (1999), Lin et. al. (2004)) that ignore these effects.

The net effect of the *zoning-interaction*, *targeting*, and *multiplier effects* under our benchmark case is to increase total improvements by 26% relative to the absence of zoning. Because zoning is most binding in neighborhoods with high-density and high levels of housing deterioration, zoning re-directs revitalization funds from these neighborhoods to low-density neighborhoods with moderate and low levels of housing deterioration. While zoning causes the equilibrium g to rise to balance the government's budget constraint (2.18), the set of landlords who now make improvements have lower housing densities, resulting in a net lower marginal cost of the subsidy to the government. In the example presented here, the increased cost of subsidies due to the increased per-unit subsidy g is more than offset by the fact that zoning induces lower-density neighborhoods to increase their improvements. This leads to an increase in aggregate improvements of 26%. This spatial reallocation of subsidies is crucial when assessing policy impacts across spatially-connected, heterogeneous neighborhoods.

4.2. Changes in Housing Supply across Space

We now examine changes in housing supply across the city under a housing revitalization subsidy in the absence and presence of zoning under our benchmark

assumptions, where zoning stringency is still set to 17,500 sq. ft. Under this stringency, zoning is binding up to 1.5 miles, about halfway to the edge of the central city area. In figure 2, the horizontal axis measures distance, in miles, from the CBD. The vertical axis measures the change in housing supply per unit of land due to the housing revitalization subsidy. The goal of figure 2 is to decompose the overall housing market response due to interactions of zoning with a housing revitalization subsidy.

$\Delta h^{NZ}(x)$ and $\Delta h^Z(x)$ represent the change in housing supply due to the housing revitalization subsidy in the absence and presence of zoning, respectively. The area under each curve represents the change in total housing supply. In the absence of zoning, increases in housing supply have an inverted U-shape between 0.9 and 2.1 miles from the CBD because these are the neighborhoods where improvements take place (see figure1). Increases in housing supply outside these neighborhoods result from the increase in total improvements, which are capitalized into housing prices. This stimulates housing construction in the rest of the central city area and in other parts of the city (namely between 3.5 and 5 miles from the CBD).

Compared to the no zoning case, zoning sacrifices 10% of housing supply in neighborhoods where zoning is binding up to 1.5 miles from the CBD. On the other hand, zoning increases housing supply within the unconstrained neighborhoods of the central city area (located between 1.5 and 3.5 miles) by 0.1% due to the increase in total improvements. Because the increase in total improvements also spills over to adjacent neighborhoods and is capitalized into housing prices, housing supply up to 1 mile from the central city area increases 0.04% compared to the no zoning case, which is consistent with empirical studies such as Schwartz et al. (2006). Finally, suburban housing supply

(from 4.5 miles on) decreases 0.03% as the quality of inner-city neighborhoods increases due to zoning. The net effect is to decrease overall housing supply in the city by 1.7% and shift housing supply away from the CBD and towards the suburbs.

4.3. The Significance of Zoning stringency

In the previous subsections, we presented a case where the *targeting effect* dominated the *zoning-interaction effect*, leading to an increase in total housing improvements of 26% and a decrease in total housing supply of 1.7%. In order to further examine the significance of zoning on the effectiveness of a housing revitalization program we vary the zoning stringency between 0 sq. ft and 27,500 sq. ft. Results are summarized in table 2 and are expressed as percentage changes compared to the no zoning case.

The crucial point from table 2 is that the *targeting effect* dominates the *zoning-interaction effect* only for low-to-moderate zoning stringencies. As the stringency of zoning increases, the loss from the *zoning-interaction effect* increases. On the other hand, the gains in improvements from the *targeting effect* increase up to a stringency of 22,500 square feet, and then start to decline. Initially, zoning re-allocates funding from high-density areas to low-density neighborhoods, increasing total improvements. The eventual decline in total improvements occurs as zoning starts restricting improvement decisions by most landlords in the central city area. As a result, it starts to be very costly for government to subsidize housing revitalization. This is not surprising since zoning is an indirect instrument to geographically target revitalization funds. Note that the *multiplier effect* merely magnifies the positive or negative net effects of the *zoning-interaction* and *targeting effects*.

Moreover, zoning reduces housing supply in the most central parts of the city (neighborhoods located between 0 and 1.7 miles from the CBD) by up to 23%. However, provided total improvements increase, housing supply will also increase whenever zoning is not binding (neighborhoods located between 1.7 miles and 5 miles from the CBD). Nonetheless, for the parameters considered here, zoning always results in a net decrease in total housing supply.¹⁸

4.4. Further Sensitivity Analysis

We end this section by exploring the sensitivity of the above results to some additional key parameters. We focus on the cases where a change in the parameters would likely affect the magnitude of the effects, and we include the benchmark as the reference case. In particular, we vary the initial level of housing improvements, the degree and spatial distribution of low quality housing, and the total amount of housing revitalization subsidies. The change in the degree of low quality housing is captured through changes in the size of the central city area. Table 3 summarizes the results of this sensitivity analysis.

Varying the size of the central city area: In the second row in table 3, we vary the extent of the central city area between 2.5 miles (low) and 4.5 miles (high).¹⁹ Increasing the size of the central city area increases the number of households with deteriorated housing, which increases the effectiveness of a \$4 million housing revitalization subsidy to increase total improvements. Total improvements were 30.07 imp/sqft in the central

¹⁸ This is consistent with empirical findings such as Mayer and Sommerville (2000) who find a consistent reduction in housing supply due to increased regulation. The elasticity of supply with respect to stringency in our model is -0.15.

¹⁹ In the central case, we have assumed that the extent of low quality housing was 3.5 miles in radius from the CBD.

case, and vary between 14.70 and 64.34 in the low and high scenarios. Note also that as the size of the central city area increases, the equilibrium subsidy per unit of improvement decreases: \$0.0157/imp/sqft (low), \$0.0138/imp/sqft (central) and \$0.0132/imp/sqft (high). In excessively declined cities with large areas of low quality housing, equilibrium subsidies are lower, as the private marginal benefits of improvements are increased. In less declined cities, there are fewer neighborhoods with larger marginal benefits, increasing the size of the equilibrium per unit subsidy.

Varying the total amount of housing revitalization subsidies: In the third row of table 3, we vary the amount of revitalization subsidies between \$2 million (low) and \$8 million (high). For our central case we assumed a \$4 million subsidy. Not surprisingly, as the amount of housing subsidies increases, both total improvements and total housing supply also increase. However, they increase at a decreasing rate as the gains from the *targeting effect* start to dissipate. For example, when total subsidies double from \$2 million to \$4 million, improvements increase by 109% and when total subsidies double from \$4 million to \$8 million, improvements increase by 107%.²⁰

Varying the spatial location of low quality housing and the initial level of improvements: Finally, we relax two of our initial assumptions regarding the spatial distribution of low quality housing and the initial level of improvements. When we reduce the price of housing improvements to the level where private landlords make improvements in the absence of a subsidy, we find that improvements become

²⁰ We also varied the total amount of subsidies from \$4 million to \$6 million and from \$6 million to \$8 million, which implied 52% and 36% increases in total improvements, respectively.

concentrated near the CBD.²¹ Instead of an inverted U-shape (figure 2), improvements across space become triangular with a peak at the CBD when we introduce a subsidy. Consistent with the analytical model, an increase in zoning in this case reduces the incentives for landlords near the CBD to make improvements, via the *zoning-interaction effect*, but also leads to an increase in subsidy via the *targeting effect*. Because housing supply tends to be greatest near the CBD, the *zoning-interaction* effect is stronger than in the benchmark, and we find that the two effects roughly offset each other.

We also relax our assumption regarding the spatial distribution of the physical deterioration of the housing stock by setting housing deterioration to be zero at the CBD and letting it rise towards the central city area boundary. In this case, the spatial distribution of housing improvements becomes concentrated near the boundary of the central city area (3.5 miles). Here we find that the *targeting effect* is much weaker than in the benchmark, such that zoning leads to a decrease in overall improvements. Space plays an important role in limiting the positive *targeting effect*, because in contrast to the benchmark, there are no unzoned neighborhoods making little-to-no improvements in equilibrium. Thus, the unzoned neighborhoods that could take advantage of the increase in the subsidy have already exhausted most of the private benefits of making improvements.

5. Conclusions

In this paper, we have examined how zoning impacts the effectiveness of housing revitalization subsidies in terms of increasing total housing improvements. Our analytical

²¹ Because landlords near the CBD have a larger housing supply to improve, they are more sensitive to marginal costs per square foot of improvements. Lowering that cost makes them relatively more willing to make improvements compared to landlords with smaller stocks further from the CBD.

results show that zoning has two opposing effects. On the one hand, zoning decreases the marginal return to housing improvements for zoned landlords, and thus their willingness to make improvements (*zoning-interaction effect*). On the other hand, by reducing housing improvements in some neighborhoods, additional funds are allocated to other neighborhoods within the city who respond by increasing improvements (*targeting effect*).

Our simulation analysis has presented a case where zoning increases the effectiveness of housing improvement subsidies up to 26%. This occurred when low quality housing was located in high density areas (where the cost of housing improvements is very high), and zoning shifted housing subsidies away from this area towards neighborhoods with moderate density, increasing total improvements generated by the same amount of subsidy. However, this increase in the effectiveness of housing revitalization subsidies is contingent upon the initial level of zoning. In fact, the positive *targeting effect* only dominates the *zoning-interaction effect* for moderate levels of zoning stringency. As the stringency of zoning increased, revitalization funds shifted towards neighborhoods with low levels of deterioration, where marginal benefits of housing improvements were very small. Consequently, the subsidy per unit of housing improvement had to be higher to encourage improvements in these neighborhoods and as such, total housing improvements generated with the same amount of revitalization funds were lower.

Further sensitivity analysis also shows that when low quality housing was located in low density areas, the presence of zoning reduced total housing improvements. In this case the presence of zoning was not only a barrier in terms of housing improvements, but also a significantly reduced housing supply. Thus, our results suggest that policymakers

should take into account how spatial heterogeneity in terms of housing supply, existing structural density and location of low quality housing vary across communities when evaluating whether or not zoning may be a barrier to the effectiveness of housing revitalization subsidies. In addition, our results are also consistent with the idea that not all neighborhoods should be targeted for a housing revitalization subsidy (Accordino et al. (2005), Rossi-Hansberg (2008)). In our analysis, neighborhoods with either very high marginal costs of improvements or very low marginal benefits of improvements had the lowest total improvements generated with the same amount of revitalization funds.

One shortcoming of our analysis is that it does not explore the interrelationship between public and private funding for moderate housing revitalization. In particular, we do not disentangle the crowding out effect of subsidized improvements on private improvements. As a result, we do not explore how zoning may impact crowding out effects of housing revitalization subsidies. In addition, by ignoring the presence of building codes, our analysis may overstate the gains from preexisting zoning in blighted urban areas. Because building codes are generally written for new construction with little emphasis on revitalization work, they can compromise revitalization efforts of blighted areas. If so, the presence of outdated revitalization codes can mitigate the positive *targeting effect*.

Finally, though the goal of these policies (and thus our analysis) focuses on the effectiveness of housing revitalization subsidies, a natural extension would be to consider the welfare effects of housing revitalization subsidies in the presence of zoning. Because improvements are capitalized into housing prices, if zoning increases improvements we would see a positive welfare effect. On the other hand, an increase in zoning reduces

housing density, creating a negative welfare effect. Though not presented here, additional simulation analyses looking at the welfare effects (measured by changes in aggregate property values) of increasing revitalization subsidies with and without zoning found a positive welfare effect in the presence of zoning. This suggests that the aggregate gain in capitalization of improvements can in fact outweigh the loss from reduced housing density.

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Appendix A: Analytical Derivations

Deriving equation (2.14)

The first-order condition for optimal housing improvements in the presence of a housing revitalization subsidy is given by:

$$\frac{\partial p(x)}{\partial I(x)} h(S(x)) - \frac{\partial C^I(x)}{\partial I(x)} + gh(S(x)) = 0 \quad (\text{A.1})$$

Applying the Implicit Function Theorem to (A.1) while keeping g and \bar{I} fixed yields (2.14).

Deriving equation (2.15)

Differentiating the government budget constraint (2.7) with respect to $\hat{\lambda}$, holding feedbacks of aggregate improvements into individual improvements constant ($dI^*(x)/d\bar{I} = 0$) and taking equation (2.14) and the equilibrium housing revitalization subsidy $g(y, t, \bar{I}, \bar{V}, \hat{\lambda}, \delta(x), M)$ into account yields:

$$\frac{\partial g}{\partial \hat{\lambda}} \int_0^{\bar{x}} I(x) h(S(x)) 2\pi x dx + g \int_0^{\bar{x}} \left[\frac{\partial I(x)}{\partial \hat{\lambda}} + \frac{\partial I(x)}{\partial g} \frac{\partial g}{\partial \hat{\lambda}} \right] h(S(x)) 2\pi x dx = 0 \quad (\text{A.2})$$

Finally, solving (A.2) for $\frac{\partial g}{\partial \hat{\lambda}}$, yields equation (2.15).

Deriving equation (2.16)

Totally differentiating (2.12) with respect to $\hat{\lambda}$, while taking into account that the equilibrium housing revitalization subsidy is given by $g(y, t, \bar{I}, \bar{V}, \hat{\lambda}, \delta(x), M)$, yields:

$$\frac{d\bar{I}}{d\hat{\lambda}} = \int_0^{\bar{x}} \frac{\partial I(x)}{\partial g} \frac{dg}{d\hat{\lambda}} 2\pi x dx + \int_0^{\bar{x}} \frac{\partial I(x)}{\partial \hat{\lambda}} 2\pi x dx + \int_0^{\bar{x}} \frac{\partial I(x)}{\partial \bar{I}} \frac{d\bar{I}}{d\hat{\lambda}} 2\pi x dx \quad (\text{A.3})$$

Solving (A.3) with respect to $\frac{d\bar{I}}{d\hat{\lambda}}$ yields (2.16).

Appendix B – Structure of the Numerical Model

Note that for calibration purposes, the utility function in the numerical model is calibrated in terms of *disutility*, $D(x)$, from low-quality housing. The underlying mechanics and relationships described in the analytical model are nonetheless unchanged.

Functional Forms of the Numerical Model

Households' preferences are represented by a Cobb-Douglas utility function:

$$U(H(x), Z(x), D(x)) = H(x)^\alpha Z(x)^{(1-\alpha)} (1 - \nu D(x)^\theta)$$

(B.1)

where α denotes the percentage of income net of transportation costs spent on housing; ν represents household valuation of housing quality; and θ is the elasticity of utility with respect to housing quality.

The subutility for housing quality is represented by a C.E.S. function:

$$D(x) = \left[Q(x)^{1-1/\varphi} + A(x)^{1-1/\varphi} \right]^{1/(1-1/\varphi)} \quad (\text{B.2})$$

where φ is the degree of complementarity between private and aggregate improvements.

The physical deterioration of the stock is given by:

$$Q(x) = \underline{Q} \left[\frac{x_B - x}{x_B} \right]^\delta - I(x) \quad (\text{B.3})$$

where δ is the rate at which physical deterioration decreases with distance and x_B is the extent of the central city area of low housing quality. The spillover effects of this low quality housing area are computed as:

$$A(x) = \frac{1}{\pi x_B^2} \int_0^{x_B} Q(z) 2\pi z dz * \begin{cases} 1 & \text{if } 0 \leq x \leq x_B \\ \frac{e^{-\rho(x-x_B)} - \xi}{1-\xi} & \text{if } x_B \leq x \leq \bar{x} \end{cases} \quad (\text{B.4})$$

where ρ is the rate at which the negative spillovers decrease with distance from x_B , and ξ is a parameter that captures household valuation for suburban residence.

The housing floor area produced per unit of land is described by a CES function:

$$h(S) = \left[\beta S^{\frac{\sigma-1}{\sigma}} + (1-\beta) \right]^{\frac{\sigma}{\sigma-1}}, \sigma \neq 1 \quad (\text{B.5})$$

where β is the share of capital in the production of housing and σ is the elasticity of substitution between capital and land in the production of housing.

Total costs of improvements are quadratic and convex, following Hall (2004):

$$C^I(S(x), I(x)) = p_I h(S(x)) I(x) + \omega h(S(x)) I(x)^2 \quad (\text{B.6})$$

where ω is the improvement cost adjustment factor per square feet of housing stock and p_I is the price of improvements per square feet of housing stock.

Construction costs are linear and given as:²²

$$C^S(S(x)) = p_k S(x) \tag{B.7}$$

where p_k is the per-unit price of capital.

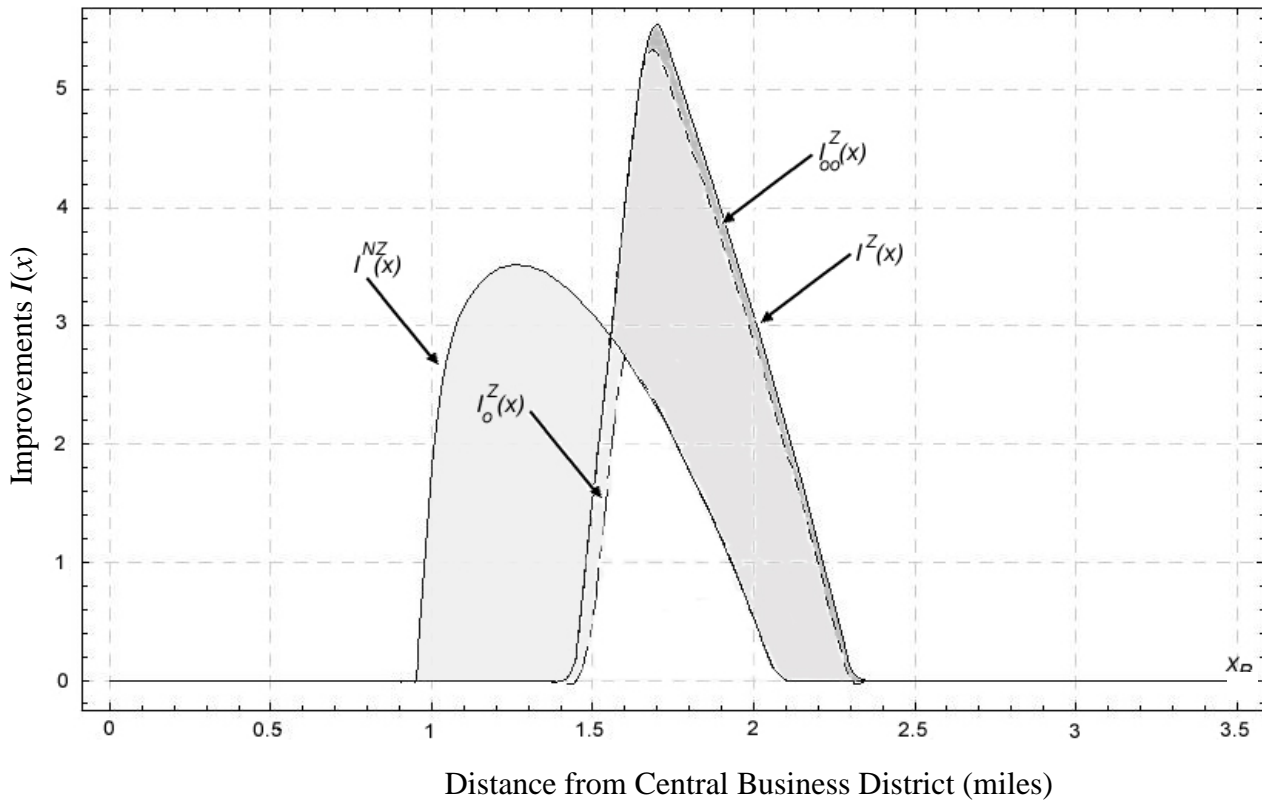


Figure 1: Improvements in the central city area with and without zoning

²² The R.S. Mean data (2000) suggests the marginal construction cost of an apartment, which is the price of building up, to be flat within a wide range of heights above seven stories.

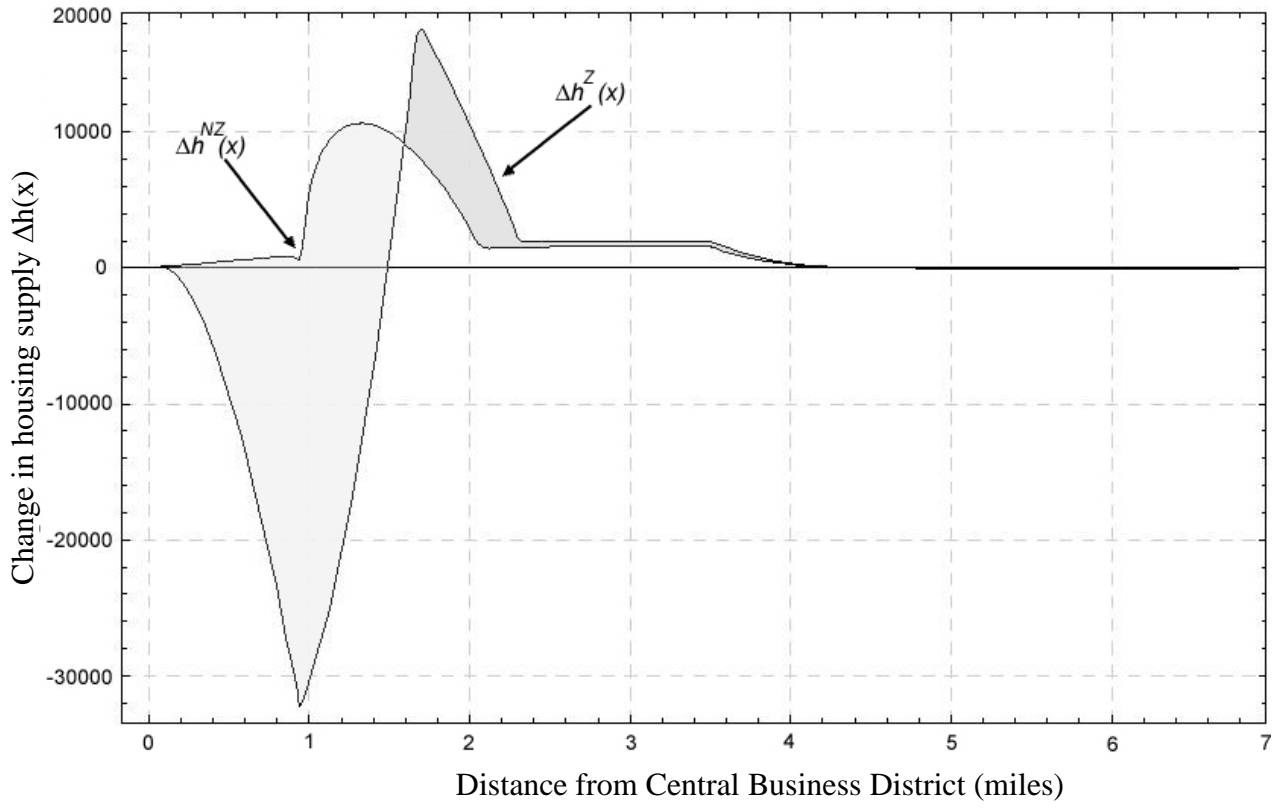


Figure 2: Change in housing supply with and without zoning

Table 1: Numerical model parameters

Parameter Description	Parameter	Value
Household income	\bar{y}	\$40,000
Transportation cost per mile (annual)	t	\$600
Percentage of income net of transportation costs spent on housing	α	0.4
Elasticity of utility with respect to housing quality	θ	2
Household valuation of housing quality	ν	0.000004
Geographic extent of the low-quality housing	x_B	3.5 mi.
Rate at which negative externalities decrease with distance	δ	0.5
Maximum quality of the housing stock	\underline{Q}	100
Rate at which negative spillovers decrease with distance	ρ	1
Complementarity parameter	φ	-1
Suburban preference parameter	ξ	0.333
Elasticity of substitution between capital and land	σ	0.8
Share of capital in the production of housing	β	0.996
Price of capital per unit of capital	p_k	\$9
Price of improvements per square feet of existing housing	p_I	\$0.03
Improvements cost adjustment factor per square feet of existing housing	ω	\$0.000005
Exogenous utility level	\bar{V}	7380

Table 2: Significance of Preexisting Zoning

Zoning stringency [0sqft-10,000sqft]	% Change relative to the no zoning case					% Change relative to the no zoning case				
	Per-unit subsidy	Community quality	Zoning interaction effect	Targeting effect	Multiplier effect	Total Housing Supply	Total Housing Supply across space			
							[0mi-1.7mi]	[1.7mi-3.5mi]	[3.5mi-5mi]	[5mi-7mi]
	0%	0%	n.a	n.a	n.a	0%	0%	0%	0%	0%
12,500sqft	0.04%	3.40%	-4.14%	5.60%	1.94%	-0.55%	-2.43%	0.01%	0.01%	0.00%
15,000sqft	0.41%	10.77%	-29.09%	35.84%	4.01%	-1.04%	-4.85%	0.37%	0.02%	-0.01%
17,500sqft	1.29%	26.18%	-58.97%	78.16%	6.99%	-1.74%	-8.78%	1.31%	0.04%	-0.03%
20,000sqft	2.90%	42.96%	-100%	134.50%	8.46%	-2.81%	-14.01%	2.05%	0.07%	-0.05%
22,500sqft	5.61%	58.07%	-100%	150.29%	7.78%	-4.32%	-18.32%	0.00%	0.10%	-0.06%
25,000sqft	9.20%	49.46%	-100%	145.28%	4.18%	-6.46%	-21.32%	-6.07%	0.08%	-0.05%
27,500sqft	12.86%	-5.19%	-100%	95.02%	0.21%	-9.28%	-22.67%	-16.56%	-0.01%	0.01%

Table 3: Further Sensitivity Analysis

		Per-unit subsidy (\$/I(x)/sqft)	Total Improvements (I(x)/sqft)	Total Housing Supply (sqft)	Total Housing Supply (sqft) across space			
					[0mi-1.7mi]	[1.7mi-3.5mi]	[3.5mi-5mi]	[5mi-7mi]
Central case		\$0.0138	30.07	1,221,700	268,672	489,845	330,869	132,328
Extent of low-quality housing	Low	\$0.0157	14.7	1,380,102	269,020	619,223	355,329	136,548
	High	\$0.0132	64.34	1,070,200	260,019	430,648	252,771	126,763
Total housing rehabilitation subsidies	Low	\$0.0137	14.34	1,212,240	267,850	485,392	330,583	132,419
	High	\$0.0140	62.35	1,231,830	269,020	499,214	331,451	132,144