Contagion of Financial Crises in Sovereign Debt Markets

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Abstract

This paper develops a DSGE model of sovereign default and contagion of financial crises for small open economies that have common risk averse international investors. The paper extends the literature on endogenous default risk to the case in which several emerging economies that cannot credibly commit to honor their international debts have common investors. The existence of common investors with preferences that exhibit decreasing absolute risk aversion (DARA) generates financial links between the emerging economies sovereign debt markets that help to explain the endogenous determination of credit limits, capital flows, and the risk premium in sovereign bond prices as function not only of the economy's fundamentals, the investors' characteristics - wealth, and degree of risk aversion - but more importantly of the fundamentals of other emerging economies. The model formalizes theoretically the existence of two important channels of contagion that are often cited in the empirical literature on the subject: "The wealth channel of contagion", and "The Portfolio Recomposition Channel of Contagion". Quantitatively, the model is consistent with the observed episode of contagion of the Argentinean crisis to its neighbor country Uruguay.

Keywords: Contagion; Sovereign Default Risk; Financial Links; Default; Flight to Quality
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1 Introduction

In the last decades the world has witnessed several financial crises that have occurred simultaneously across countries. Examples of these simultaneous crises are the Debt Crisis of 1982, the Mexican Crisis of 1994, the Asian Crises in 1997, the Russian Crisis of 1998, and more recently the Euro-debt Crisis in 2011. While crises could be simultaneous because of the occurrence of a common shock to several economies another plausible explanation for these simultaneous crises is contagion. Contagion corresponds to the transmission of a negative income or financial shock that occurs in one economy to other economies. The empirical literature that looks at the simultaneity of crisis is quite large, and evidence of contagion in sovereign bonds markets is considerable.[1]

The current paper is concerned with advancing an endogenous theory of contagion of financial crises based on financial links between economies.[2] The emphasis on financial links between economies as the channel for contagion is based on the findings of several empirical papers that have studied the role of financial links in explaining contagion.[3]

The model in the paper studies financial market links across countries in a dynamic stochastic general equilibrium setting where the stochastic processes of the emerging economies' bond prices are endogenously determined. The model extends the literature in endogenous sovereign risk in order to consider sovereign bond markets in a multi-country framework.[4] This type of model allows for an endogenous determination of the price of one period non-contingent discount bonds as a function of the economy's default risk. Through the consideration of financial links across economies, the default risk of any economy becomes a function not only of the domestic fundamentals but also a function of investors characteristics, and the fundamentals of countries which share investors with the domestic country. The model is used to show quantitatively that contagion can explain co-movements in the price of emerging economy bonds, capital flows, output and consumption beyond the level explained by a country's own fundamentals.

Within the present model, the framework is one of a set of small open economies with

[1]See for example Valdes (1996), Baig and Goldfajn (1998), Edwards (1998), and Baig and Goldfajn (2000).

[2]Countries are linked financially when they have common investors.

[3]See for example Kaminsky and Reinhart (1998), Van Rijckeghem and Weder (1999), Kaminsky et al.(1999), Kaminsky and Reinhart (2000), Hernandez and Valdes (2001).

[4]Some papers in the literature of endogenous sovereign risk are Aguiar and Gopinath (2006), Arellano (2008), Bai and Zhang (2006), Cuadra and Sapriza(2008), Lizarazo (2011),Hatchondo, Martinez and Sapriza (2008), Martinez and Hatchondo (2009), Mendoza and Yue (2008), and Yue (2006).

stochastic endowments. These small open economies have access to an international credit market populated by international investors. International investors are assumed to be risk averse, with preferences that exhibit a decreasing absolute risk aversion in wealth (DARA). Due to the fact that the number of countries is finite, international investors are not able to completely diversify the risk of their investment in any country. There is a problem of enforcement in the sense that international investors cannot force the small open economies to repay their debts. If any economy defaults it is temporarily excluded from the world asset market. Countries weight the benefits and costs of default, and decide to repay or not. This context forces international investors to consider the risk of default when choosing their portfolio. Any type of reallocation of the international investors portfolio has effects over several countries at the same time, therefore the risk of default is endogenously determined by the own economy fundamentals, and by the fundamentals of other economies: income shocks to an emerging economy generate changes in the risk of default in that economy, and through financial links, these changes in turn impact other emerging economies. Financial links generate contagion through two channels, the *Wealth* channel , and the *Portfolio Recomposition* channel.

(i) The **Wealth** channel of contagion: When an income shock in a country forces this country into default, the shock to the first country generates losses for international investors. If the preferences of the investors exhibit DARA the negative wealth effect of the shock reduces investors' tolerance for risk. A reduction in tolerance for risk makes investors shift away from risky investments (countries) toward riskless (T-Bills). Countries that did not default or face an income shock would face a reduction in the amount of resources available to borrow from, and contagion occurs.

(ii) The **Portfolio Recomposition** channel of contagion**:** When the risk of default is correlated across countries, an increase in the risk of default in the first country generates, all other things given, an increase in the overall risk of the investors' portfolio as investors expect other countries risk of default to also increase. In this case, the change in the risk of default in one country modifies the optimal portfolio of international investors. As investors adjust their portfolios, countries which did not face an income shock nonetheless face a reduction in the amount of resources available to borrow from, and contagion occurs.[5]

[5]In this model the risk of default across countries is correlated because of the possibility of contagion caused by financial links manifested through the previously explained wealth channel.

The the wealth channel of contagion is analyzed in the papers of Goldstein and Pauzner (2001), Kyle and Xiong (2001), and Lagunoff and Schreft (2001), these papers show that if investors' preferences exhibit decreasing absolute risk aversion (DARA) the optimal response of the investors to financial losses is to reduce their risky investments, as consequence of the reduction on their tolerance toward risk at lower levels of wealth. The portfolio recomposition channel of contagion is studied in the theoretical papers of Choueri (1999), Schinasi and Smith (1999) and Kodres and Pritsker (2002). Using a static partial equilibrium approach where the determination of asset returns is exogenous to the model, these papers highlight the fact that contagion might be successfully explained by standard portfolio theory: in order to reestablish the optimal degree of risk exposure in their portfolio after a negative shock to the return of the assets of some economy, it is optimal for investors to liquidate their holdings of assets with expected returns that exhibit some correlation with the expected return of the crisis country—whether this correlation is a consequence of contagion or just correlated fundamentals.

The results of this paper are consistent with the empirical evidence regarding contagion as consequence of financial links. First, since investors' preferences exhibit DARA they are able to tolerate more default risk when they are wealthier. As consequence both capital flows to emerging economies, and the equilibrium price of sovereign bonds are increasing functions of investors' wealth levels. In turn, this result implies that the current model can explain the high correlation of sovereign bond spreads and capital flows across emerging economies. Furthermore, the high correlation between investors' wealth and emerging economies financing conditions can account for the simultaneity of crises because a default by any economy is equivalent to a negative wealth shock to the investors. This shock is transmitted to other countries via the wealth channel of contagion.

Second, because of the common investors, when the probability of default increases for some foreign country other countries' financing conditions change. When the probability of default for some foreign country increases, two opposing forces affect the financing situation of other emerging economies: On the one hand, a decrease in the price of the sovereign bonds of the foreign country constitutes an expected future negative wealth shock to the investors due to the higher associated probability that this country will default, increasing the default risk of the other economies. On the other hand, an increase in default probabilities induces a substitution away from the assets of the economies whose risk responds more strongly towards the assets of the economies whose risk did not increase too much. This effect would tend to increase the set of financial contracts available to some emerging economies. For any country different from the crisis country, if the first effect dominates contagion is observed: the correlation of capital flows across emerging economies is positive. In the other hand, if the second effect dominates, "flight to quality" is observed: emerging economies with robust fundamentals receive capital flows when several other countries are affected by financial crises. Whenever investors' wealth is *sufficiently* low, or their degree of risk aversion is *sufficiently* high, or the economies fundamentals are *sufficiently* weak, the effect of the expected negative wealth shock will dominate the substitution.

Third, the likelihood of default in equilibrium for any emerging economy is a function not only of investors' characteristics and the economy's own fundamentals, but also of other emerging economies' fundamentals. In the numerical simulations in the present paper, default is more likely to be an equilibrium outcome when the fundamentals of other economies deteriorate.

The paper proceeds as follows: Section II develops the model; section III characterizes the equilibrium of the model; section IV presents the numerical results of the paper; and section V concludes.

2 The Model

Before going into the description of the model it is useful to define the state of the world S in the model:

Definition 1 *The state of the world, $S = (s, W)$, is given by the realization of the emerging economies' fundamentals, $s = s_1 \times s_2 \times \ldots \times s_J$ and the representative investor's asset position or wealth, W. In this model $s_j = (b_j, y_j, d_j)$ where b_j is economy's j asset position, y_j is economy's j endowment, and d_j is a variable that describes if the economy j is in default or repayment state.*

2.1 Sovereign Countries

There are $J < \infty$ identical small open economies populated each of them by risk averse households that maximize their discounted expected lifetime from consumption

$$\max_{\{c_{j,t}\}_{t=0}^{\infty}} \qquad E_\tau \sum_{t=0}^{\infty} \beta^t u\left(c_{j,\ t}\right) \tag{1}$$

where $0 < \beta < 1$ is the discount factor and $c_{j,\ t}$ is the j emerging economy's consumption at time t. The j emerging economy's periodic utility takes the functional form

$$u(c_j) = \frac{c_j^{1-\gamma}}{1-\gamma}$$

where $\gamma > 0$ is the coefficient of relative risk aversion.

In each period, the households of each economy j receive a stochastic stream of consumption goods y_j. This income is independently distributed across emerging economies. Also, this income is non-storable, its realizations are assumed to have a compact support; the stream of income follows a Markov process drawn from probability space $(y_j, Y_j(y_j))$ with a transition function $f(y'_j \mid y_j)$. Households in each economy j also receive a lump-sum transfer from their government.

The government of each economy j is a benevolent government that aims to maximize the lifetime utility of the households in the economy. The governments of the economies have access to international financial markets, where they can trade one-period non-contingent bonds with international investors.[6]. The governments of the economies use their access to financial markets to smooth the consumption path of the households in their economy.

In the international financial markets the governments borrow or save by buying on period bonds b'_j at price $q_j(b'_j, S)$. Both the investors and the government of each economy j take as given the price of the emerging economy j non-contingent discount bonds. In each period, the government of economy j rebates back to the households of the economy j all the proceedings from its international credit operations in a lump-sum fashion.

The bonds of any emerging economy j, b'_j are risky assets because debt contracts between the government of any emerging economy j and the investors are not enforceable. At any time, the government of the emerging economy j can choose to default on its debt. If the government of country j defaults, all its current debt is erased, and this government is temporarily excluded from international financial markets. Defaulting also entails a direct output cost for country j.

Because international investors are risk averse, the bond prices of the emerging economies, $q_j(b'_j, S)$ for $j = 1, \ldots, J$, have two components: the price of the expected losses from default $q_j^{RN}(\delta_j(b'_j, S))$ that corresponds to the price of riskless bonds (hereafter

[6]Through out the paper it is assumed that the governments of the economies are not able to trade financial assets between them

T-Bills), q^f, adjusted by the default probability of the economy j, $\delta_j(b'_j, S)$,and an "excess" premium or risk premium $\zeta_j^{RA}(b'_j, S)$. This result will be discussed in more detail in the next sub-section.

For any emerging economy j, when $b'_j \geq 0$, the probability of default by the government of the economy, $\delta_j(b'_j, S)$, is 0, and since the asset is riskless in this case, the risk premium, $\zeta_j^{RA}(b'_j, S)$, is also 0. Therefore the price of the bond of the emerging economy j is equal to the price of T-Bills which is $q^f = \frac{1}{1+r}$, where r is the constant international interest rate. Only when $b'_j \leq 0$ can $\delta_j(b'_j, S)$ and $\zeta_j^{RA}(b'_j, S)$ be different from 0.

For any economy j when its government chooses to repay its debts, the resource constraint of the emerging economy is given by

$$c_j = y_j - q_j(b'_j, S)b'_j + b_j, \tag{2}$$

For the same economy, when the government chooses to default the resource constraint of the emerging economy is given by

$$c_j = y_j^{def}, \tag{3}$$

where $y_j^{def} = h(y_j)$ and $h(y_j)$ is an increasing function.

The timing of decisions within each period on the side of any emerging economy j is as follows: the government of the economy starts with initial assets b_j, observes the income shock y_j, and the income shocks to all other economies in the representative international investor's portfolio $\{y_k\}_{k=1,k\neq j}^{J}$, and decides whether to repay its debt or to default: all economies $j = 1, \ldots, J$ make this decision at the same time. If the government of economy j decides to repay, then taking as given the bond price schedule $q_j(b'_j, S)$, this government chooses its next period asset position b'_j subject to the resource constraint. Finally consumption of the emerging economy's households c_j takes place.

Define $V_j^0(S)$ as the value function of the government of economy j that has the option to default. The government starts the current period with assets b_j and income y_j, the other economies that share investors with country j start the current period with assets b_k and income y_k for $k = 1, \ldots, J$, and $k \neq j$, all these countries face a representative international investor that has wealth W. The government of any economy j decides wether to default or repay its debts to maximize the households' welfare. This government takes as given the decisions of repayment or default of other governments regarding their debts with the representative international investor. Given the option of default for country j, $V_j^0(S)$

satisfies

$$V_j^0(S) = \max_{\{R,D\}} \left\{ V_j^R(S), V_j^D(y_j, \{s_k\}_{k=1,k\neq j}^J, W) \right\} \tag{4}$$

where $V_j^R(S)$ is the value to the government of country j of repaying its debt and $V_j^D(y_j, \{s_k\}_{k=1,k\neq j}^J, W)$ is the value of defaulting in the current period.

If the government of economy j defaults the value of default is given by

$$\begin{aligned} V_j^D(y_j, \{s_k\}_{k=1,k\neq j}^J, W) \;&=\; u(y_j^{def}) \;+ \\ \beta \int_{y_1'} \cdots \int_{y_J'} [\theta V_j^0(0, y_j', \{s_k'\}_{k=1,k\neq j}^J, W') \;&+\; (1-\theta) V_j^D(y_j', \{s_k'\}_{k=1,k\neq j}^J, W')] \prod_{h=1}^J f(y_h, y_h') dy_h'. \end{aligned}$$

where θ is the probability that any defaulting economy regains access to credit markets.

If the government of economy j repays its debts, the value of not defaulting is given by

$$V_j^R(S) = \max_{\{b_j'\}} \left\{ u(y_j - q_j(b_j', S) b_j' + b_j) + \beta \int_{y_1'} \cdots \int_{y_J'} V_j^0(S') \prod_{h=1}^J f(y_h, y_h') dy_h' \right\}.$$

For the government of the emerging country j the decision of default/repayment depends on the comparison between the value of repaying its debt, $V_j^R(S)$, versus the value of opting for financial autarky, $V_j^D(y_j, \{s_k\}_{k=1,k\neq j}^J, W)$. The repayment/default decision of the government is summarized by the indicator variable d. In the context of this model, when the government pays back its debt this variable takes the value of 1 and when the government does not pay back this variable has the value of 0. The functional form of the default/repayment decision is given by

$$d_j = \left\{ \begin{array}{ll} 1 & \text{if } V^R(S) > V^D(y_j, \{s_k\}_{k=1,k\neq j}^J, W) \\ 0 & \text{otherwise} \end{array} \right\}$$

This repayment/default decision is a period by period decision.

It is also important to notice that the government of economy j faces a lower bound on debt $\underline{\underline{B_j}} < 0$ that prevents Ponzi schemes. This lower bound on debt $\underline{\underline{B_j}}$ is not binding in equilibrium (i.e. $b_j' \geq \underline{\underline{B_j}}$).

For each economy j, and conditional on the representative investor's wealth level W, and the other economies fundamentals $\{s_k\}_{k=1,k\neq j}^J$ the emerging economy's j default policy can be characterized by its repayment and default sets:

Definition 2 *For a given level of wealth, W, and the fundamentals of other emerging economies in the investor's portfolio, $\{s_k\}_{k=1,k\neq j}^{J}$ the default set $D_j\left(b_j \mid \{s_k\}_{k=1,k\neq j}^{J}, W\right)$ consists of the equilibrium set of y_j for which default is optimal when the government's asset holdings are b_j:*

$$D_j(b_j \mid \{s_k\}_{k=1,k\neq j}^{J}, W) = \left\{y_j \in Y_j : V_j^R(S) \leq V_j^D(y_j, \{s_k\}_{k=1,k\neq j}^{J}, W)\right\}.$$

The repayment set $A_j(b_j \mid \{s_k\}_{k=1,k\neq j}^{J}, W)$ is the complement of the default set, and corresponds to the equilibrium set of y_j for which repayment is optimal when the government's asset holdings are b_j:

$$A_j(b_j \mid \{s_k\}_{k=1,k\neq j}^{J}, W) = \left\{y_j \in Y_j : V_j^R(S) > V_j^D(y_j, \{s_k\}_{k=1,k\neq j}^{J}, W)\right\}.$$

Equilibrium default sets, $D_j(b_j' \mid \{s_k'(S)\}_{k=1,k\neq j}^{J}, W'(S))$, are related to equilibrium default probabilities, $\delta_j(S' \mid S)$, by the equation

$$\delta_j(b_j', S' \mid S) = 1 - E{d_j}'(b', S' \mid S) = \int\limits_{D_j(b_j' \mid \{s_k'(S)\}_{k=1,k\neq j}^{J}, W'(S))} f(y_j' \mid y_j) dy_j' \times \prod_{k=1,k\neq j}^{J} \int_{y_k'} f(y_k' \mid y_k) dy_k'. \quad (5)$$

If the default set is empty for ${b_j}'$, then for all realizations of the economy j′s income ${d_j}' = 1$ and the equilibrium default probability $\delta_j(b_j', S' \mid S)$ is equal to 0. In this case, it is not optimal for the government of economy j to default in the next period for any realization of the economy's income. On the other hand, if the default set for b_j' includes the entire support for the economy j's income realizations, i.e., $D_j(b_j \mid \{s_k\}_{k=1,k\neq j}^{J}, W) = Y_j$, then ${d_j}' = 0$ for all realizations of the economy's income. As a consequence, the equilibrium default probability $\delta_j(b_j', S' \mid S)$ is equal to 1. Otherwise, when the default set for b_j' is not empty but does not include the whole support for the endowment realizations, $0 < \delta_j\left(b_j', S\right) < 1$.

For each economy j, associated with the default sets we can define two concepts, the *maximum credit constraint* and the *maximum safe level of debt*:

Definition 3 *For any economy j the maximum credit constraint is the maximum level of assets, $\underline{b_j}(\{s_k\}_{k=1,k\neq j}^{J}, W)$, that is low enough such that no matter what the realization of the* *income of this economy default is the optimal choice for the government of the economy. Therefore for this level of assets the default set corresponds to the entire support of the income realizations, i.e.,* $D(\underline{b_j}(\{s_k\}_{k=1,k\neq j}^J, W) \mid \{s_k\}_{k=1,k\neq j}^J, W) = Y$.

Definition 4 *For any economy j the maximum safe level of debt is the minimum level of assets $\overline{b_j}(\{s_k\}_{k=1,k\neq j}^J, W)$ for which repayment by the government of economy j is the optimal choice for all realizations of the income of this economy. For this level of assets the default set is empty, i.e.,* $D(\overline{b_j}(\{s_k\}_{k=1,k\neq j}^J, W) \mid \{s_k\}_{k=1,k\neq j}^J, W) = \emptyset$.

Because for the government of any economy j the value of repayment is monotonically decreasing in b_j, it is obvious that $\underline{b_j}(\{s_k\}_{k=1,k\neq j}^J, W) \leq \overline{b_j}(\{s_k\}_{k=1,k\neq j}^J, W) \leq 0$. $\underline{b_j}(\{s_k\}_{k=1,k\neq j}^J, W)$ and $\overline{b_j}(\{s_k\}_{k=1,k\neq j}^J, W)$ are single-valued functions.[7]

Given $\{s'_k\}_{k=1,k\neq j}^J$ and W', any investment in the emerging economy j's bonds in excess of $\underline{b_j}(\{s'_k\}_{k=1,k\neq j}^J, W')$ would imply $\delta_j(b'_j, S) = 1$. Since the default probability is one of the components of the bond prices of any economy, these investments will have a $q_j(b'_j, S) = 0$. On the other hand, all investments in the emerging economy j's bond of an amount lower than $\overline{b_j}(\{s'_k\}_{k=1,k\neq j}^J, W')$ imply $\delta_j(b'_j, S) = 0$. Because these investments are riskless it follows that $q_j(b'_j, S) = q^f$.

In this model, conditional on other economies fundamentals $\{s'_k\}_{k=1,k\neq j}^J$, and in the investors' wealth W, the well know results of comparative statics of the model of endogenous sovereign risk with risk neutral international investors follow (see for example Aguiar and Gopinath (2006) Arellano(2008)). That is: First, default sets are shrinking in the economies assets (i.e. if $b_{j,1} < b_{j,2}$ then $D_j\left(b_{j,2} \mid \{s'_k\}_{k=1,k\neq j}^J, W\right) \subseteq D_j\left(b_{j,1} \mid \{s'_k\}_{k=1,k\neq j}^J, W\right)$), therefore the probability of default $\delta_j(b'_j, S)$ is decreasing on b'_j. Second, the governments of the emerging economies only default when the economies are facing capital outflows (i.e. $b_j - q_j\left(b'_j(S), S\right) b'_j(S) < 0$). Third, conditional on the persistence of the income process not being too high, the default risk of any economy j is larger for lower levels of income. Since the economic intuition of these results is identical to the intuition in the model of endogenous sovereign default risk with risk neutral investors, it will not be discussed in detail here.

In the other hand, as in models of endogenous sovereign risk and risk averse investors

[7] The stochastic process for the endowments has a compact support, and conditional on $\{s_k\}_{k=1,k\neq j}^J$ and W, the value of the credit contract is monotonically decreasing in b_j. Monotonicity of the credit contract and compactness of the endowment support are sufficient conditions to guarantee that $\underline{b_j}(\{s_k\}_{k=1,k\neq j}^J, W)$ and $\overline{b_j}(\{s_k\}_{k=1,k\neq j}^J, W)$ are single-valued functions.

(see, for example, Lizarazo (2011)), the risk premium $\zeta_j^{RA}(b_j', S)$ is also decreasing in b_j'. Therefore bond prices $q_j(b_j', S)$ are, increasing on b_j'. This result will be discussed briefly in the next sub-section.

2.2 International Investors

There are a large but finite number of identical competitive Investors who will be represented by a representative investor. The representative investor is a risk averse agent whose preferences over consumption are defined by a constant relative risk aversion (CRRA) periodic utility function with parameter $\gamma^L > 0$. The investor has perfect information regarding the income processes of the emerging economies, and in each period the investor is able to observe the realizations of these incomes.

The representative investor maximizes her discounted expected lifetime utility from consumption

$$E_0 \sum_{t=0}^{\infty} \beta_L^t v\left(c_t^L\right) \tag{6}$$

where c^L is the investor's consumption. The periodic utility of this agent is given by

$$v(c^L) = \frac{\left(c^L\right)^{1-\gamma^L}}{1-\gamma^L} \tag{7}$$

The representative investor is endowed with some initial wealth, W_0, at time 0, and in each period, the investor receives an exogenous income X.

Because the representative investor is able to commit to honor her debt, she can borrow or lend from industrialized countries (which are not explicitly modeled here) by buying T-Bills at the deterministic risk free world price of q^f. The representative investor can also invest in non-contingent bonds of the emerging economies $j = 1, \ldots, J$ which have an endogenously determined stochastic price of $q_j(b_j', S)$. As was mentioned before, this price is taken as given by both the investor and the governments of the emerging economies.

On the side of the investor, the timing of the decisions within each period is as follows: At the beginning of the period, and before the governments have made their default/rapyment decisions, the investor observes her assets at the beginning of the period, $W^0 = \vartheta^{TB} + \sum_{j=1}^{J} \vartheta_j$, which is composed of her asset position in T-Bills, ϑ^{TB}, and her asset position in bonds of the emerging economies ϑ_j, for $j = 1, \ldots, J$. Immediately after, the shocks to the economies' income are realized, and the governments of these economies make their repayment/default decisions; at this point the investor realizes her gains/losses, and observes her actual wealth for the period, W, that is given by $W = \vartheta^{TB} + \sum_{j=1}^{J} \vartheta_j d_j$, where d_j is defined as in the sovereign countries sub-section. After observing W the investor chooses her next period portfolio allocation, investing in the economies whose governments have payed the debt from the previous period (i.e., ϑ_j' for countries with $d_j = 1$), and in T-Bills, $\vartheta^{TB'}$. Finally consumption of the representative international investor, c^L, takes place.

In each period the representative investor faces the budget constraint

$$c^L = X + W - q^f \vartheta^{TB\prime} - \sum_{j=1}^{J} q_j(b_j', S)\vartheta_j' d_j. \tag{8}$$

It is assumed that the investor cannot go short in her investments with emerging economies. Therefore whenever the governments of the emerging economies are saving, the representative international investor receives these savings and invests them completely in $\vartheta^{TB'}$. Therefore for any economy j, $\vartheta_j' = -b_j'$ if the economy is borrowing, and it is equal to 0 otherwise.[8]

The law of motion of the representative investor's wealth is given by

$$W' = \sum_{j=1}^{J} \vartheta_j' d_j'(b_j', S) + \vartheta^{TB\prime}. \tag{9}$$

Define $V_L^0(S)$ as the value function of the representative investor with assets at the beginning of the period of W^0, that faces J governments, each of them with assets b_j and income y_j at the start of the period, any of which might default:

$$V_L^0(S) = \max_{\{\vartheta_j'\}_{j=1}^{J},\ \vartheta^{TB\prime}} \left\{ v(X + W - q^f \vartheta^{TB\prime} - \sum_{j=1}^{J} q_j(b_j', S)\vartheta_j' d_j) + \beta_L \int_{y_1'} \cdots \int_{y_J'} V_L^0(S') \prod_{h=1}^{J} f(y_h, y_h') dy_h' \right\}.$$

The representative investor faces a lower bound on her asset holdings $\underline{\underline{W}} < 0$ that

[8] This assumption does not seem to be inconsistent with reality. For example, mutual funds are strictly restricted by The Investment Company Act in their ability to leverage or borrow against the value of securities in their portfolio. On the other hand, hedge funds and other types of investors face no such restrictions. Because of these regulations it seems reasonable to make the simplifying assumption that international investors are able to leverage the riskless asset, ϑ^{TB}, but must have a non-negative position in the emerging economy's asset.

prevents Ponzi schemes,

$$W' \geq \underline{\underline{W}} \tag{10}$$

$\underline{\underline{\text{W}}}$ corresponds to the "natural" debt limit discussed in Aiyagari (1994). Additionally the investors asset position in bonds of the emerging economy is non-negative, i.e. $\vartheta_j \geq 0$ for $j = 1, \ldots, J$.

The optimization problem that the representative investor faces can be described as one in which in each period, t, she optimally chooses her portfolio according to her preferences in order to maximize her discounted expected lifetime utility from consumption, subject to her budget constraint, the law of motion of her wealth, the no-ponzi condition, and the condition that $\vartheta_j \geq 0$ for $j = 1, \ldots, J$.

Because $v(c^L)$ satisfies the standard Inada conditions, and X sufficiently large, $c_L > 0$ always. Because the representative investor is not credit constrained (Equation (10)), when the government does not default in the current period the solution to the stochastic dynamic problem for this investor can be characterized by the following first order conditions:

- For $\vartheta^{TB\prime}$

$$q^f v_{c^L}\left(c^L\right) = \beta_L \int_{y_1'} \cdots \int_{y_J'} \left[v_{c^L}\left(c^{L'}\right)\right] \prod_{h=1}^{J} f(y_h, y_h') dy_h'. \tag{11}$$

- For ϑ_j'

$$q_j v_{c^L}\left(c^L\right) = \beta_L \int_{y_1'} \cdots \int_{y_J'} \left[v_{c^L}\left(c^{L'}\right)\ d_j{}'(b', S)\right] \prod_{h=1}^{J} f(y_h, y_h') dy_h'. \tag{12}$$

The set of J equations (12) determine the allocation of the resources of the representative investor to each one of the J emerging countries. Unless emerging country j is not in a default state, i.e. $d_j = 1$, emerging country j does not belong in the investment set of the international investors. If country j has not defaulted, then the jth equation(12) equates the marginal cost of allocating wealth to bonds issued by the government of the emerging country j to the discounted expected marginal benefit of this investment. The benefit of this investment is realized only in those periods in which the emerging economy j optimally chooses to repay its debts $\left(i.e., d_j{}'(b_j', S) = 1\right)$.

It is possible to manipulate equation (12) to get

$$
\begin{aligned}
q_j(b'_j, S) &= \beta_L \int_{y'_1} \cdots \int_{y'_J} \frac{\left[v_{c^L}\left(c^{L'}\right)\ d_j{}'(b'_j, s)\right]}{v_{c^L}\left(c^L\right)} \prod_{h=1}^{J} f(y_h, y'_h) dy'_h. \\
&= \beta_L \frac{Cov\left[v_{c^L}\left(c^{L'}\right),\ d_j{}'(b', S)\right]}{v_{c^L}\left(c^L\right)} + q_j^{RN}(b'_j, S). \\
&= \zeta_j^{RA}(b'_j, S) + q_j^{RN}(b'_j, S). \qquad (13)
\end{aligned}
$$

where $q_j^{RN}(b'_j, S) = q^f\left(1 - \delta_j(b'_j, S)\right)$. Equation (13) shows the two components of the bond prices of economies that trade financially with risk averse investors. The first component, $q_j^{RN}(b'_j, S)$, compensates the investors for the expected loss from default. The second component, $\zeta_j^{RA}(b'_j, S)$, corresponds to the risk premium that sovereign bonds of an economy j have to carry in order to induce risk averse investors to hold them. This term is the principal source of the transmission of crisis among countries that share investors. The main determinant of the "excess" risk premium $\zeta_j^R A(b'_j, S)$ is the covariance term in equation (13). This covariance term is non-positive: $Cov\left[v_{c^L}\left(c^{L'}\right),\ d_j{}'(b'_j, S)\right] \leq 0.$ [9]

From the set of equations (13) is possible to see that bond prices of any emerging economy are a function not only of the economy's own fundamentals but also of the investors' risk aversion and wealth level. Additionally, and more importantly for the purpose of this model, bond prices of any emerging economy are also a function of the fundamentals of other emerging economies in the investor's portfolio: Because consumption of the representative investor is a function of her wealth, her risk aversion and her investments in other economies, sovereign bond prices of the emerging economy j are also a function of those variables.

[9] If for some bond b'_j the government of the economy does not default next period in any state of the world (i.e. the default set for b'_j is empty), then $d_j{}'(b'_j, S) = 1$ for all states, and $\delta_j\left(b'_j, S\right) = 0$, $Cov\left[v_{c^L}\left(c^{L'}\right),\ d_j{}'(b'_j, S)\right] = 0$, and $q_j(b'_j, S) = q^f$. On the other hand, if for some other b'_j the government of the economy defaults next period in all states of the world (i.e. the default set for b'_j includes the complete support of the endowment realizations), then $d_j{}'(b'_j, S) = 0$ for all states and therefore $\delta_j\left(b'_j, S\right) = 1$, $Cov\left[v_{c^L}\left(c^{L'}\right), d'_j(b'_j, S)\right] = 0$, and $q_j(b'_j, S) = 0$. If $0 < \delta_j(b'_j, S) < 1$, then for the states of the word next period in which the government of the economy repays $\left[W' \mid_{d'_j(b'_j,S)=1}\right] = \vartheta'_j + \sum_{k=1,k\neq j}^{J} \vartheta'_k d'_k(b'_k, S) + \vartheta^{TB\prime}$, and for the states in which the government of the economy j defaults $\left[W' \mid_{d'_j(b'_j,S)=0}\right] = \sum_{k=1,k\neq j}^{J} \vartheta'_k d'_k(b'_k, S) + \vartheta^{TB\prime}$. Because $\left[W' \mid_{d'_j(b'_j,S)=1}\right] > \left[W' \mid_{d'_j(b'_j,S)=0}\right]$ then $\left[c^{L'} \mid_{d'_j(b'_j,S)=1}\right] > \left[c^{L'} \mid_{d'_j(b'_j,S)=0}\right]$ and by concavity of $v(\cdot)$ $\left[v_{c^L}\left(c^{L'}\right) \mid_{d'_j(b'_j,S)=1}\right] < \left[v_{c^L}\left(c^{L'}\right) \mid_{d'_j(b'_j,S)=0}\right]$, as a consequence, for b'_j with more $d'_j(b'_j, S) = 1$ $v_{c^L}\left(c^{L'}\right)$ is lower. Clearly for this case $Cov\left[v_{c^L}\left(c^{L'}\right),\ d_j{}'(b'_j, S)\right] < 0$.

In this model, conditional on other economies fundamentals $\{s_k\}_{k=1,k\neq j}^{J}$, and in the investors' realized wealth W, the results of comparative statics of models of endogenous sovereign risk with risk averse international investors follow (see Lizarazo(2011)):

Proposition 1 *For any state of the world, S, as the risk aversion of the international investor increases, the governments' incentives to default increase.*

Proof. See Appendix. ∎

Proposition 2 *Default sets are shrinking in the assets of the representative investor. For all $W_1 < W_2$, if default is optimal for b_j in some states y_j, given W_2 then default will be optimal for b_j for the same states y_j, given W_1 therefore $D_j\left(b_j \mid W_2, \{s_k\}_{k=1,k\neq j}^{J}\right) \subseteq D_j\left(b_j \mid W_1, \{s_k\}_{k=1,k\neq j}^{J}\right)$*

Proof. See Appendix. ∎

As discussed in Lizarazo (2011) the degree of investors' risk aversion is an important determinant of access of emerging economies to credit markets, and of the risk of default of the economy. In this model, the more risk averse are international investors, the higher is the default risk and the tighter is the endogenous credit constraint faced by all emerging economies. The result in Proposition 1 is consistent with empirical findings which characterize the role of investor's risk aversion in the determination of country risk and sovereign yield.[10]

Also as in Lizarazo (2011), in the present model the higher is the investor's wealth W the smaller is the default risk of any economy in the investor's portfolio, hence the more relax is the endogenous credit constraint face by such economy. Findings of several empirical papers on the literature regarding the determinants of capital flows and sovereign bonds spreads of emerging economies are consistent with the results in Proposition 2.[11] The results in Proposition 2 are also consistent with the evidence regarding financial contagion across countries who share investors. [12]

[10] see, for example, Ferruci et. al. (2004), Mody and Taylor (2004), FitzGerald and Krolzig (2003), Cunningham, Dixon and Hayes (2001), Westphalen (2001), and Kamin and von Kleist (1999) and Arora and Cerisola (2001).

[11] See, for example, Warther (1995), Westphalen (2001), Kang et al (2003), FitzGerald and Krolzig (2003), Mody and Taylor (2004), and Ferruci et al (2004).

[12] See for example Kaminsky and Reinhart (1998), Van Rijckeghem and Weder (1999), Kaminsky and Reinhart (2000) and Hernandez, and Valdes (2001).

Briefly, the intuition for these results is that, whenever investors are wealthier or less risk averse, the marginal cost for them in terms of utility of an additional unit of investment on sovereign bonds is relatively low. Taking as given the price of the bonds of an emerging economy j , it must hold that the net flow of resources to the economy $\left(\text{i.e.,} \; -q_j(b'_j, S)b'_j\right)$ is increasing in investors wealth, W and decreasing in investor's risk aversion, γ^L. Consequently, the benefits for government of the economy j of paying its debts are also increasing in W and decreasing in γ^L; therefore $\delta_j(b'_j, S)$ decreases with W, and increases with γ^L. Hence, $q_j(b'_j, S)$ for $j = 1, \ldots, J$ are increasing in W, and decreasing in γ^L.

Because both, investors' wealth, and fundamentals of other emerging economies in the investors' portfolio have an effect on the determination of bond prices of economy j, is clear that sovereign bond prices across economies that share investors are jointly determined and therefore must be correlated.

The discussion on the way in which other countries fundamentals affect the determination of economy $j's$ bond prices, and debt flows would be postponed until the section on the characterization of contagion channels.

3 Equilibrium

The recursive equilibrium for this model is defined as a set of policy functions for (i) the emerging economies' consumption $\{c_j(S)\}_{j=1}^{J}$,(ii) the governments' asset holdings $\left\{b'_j(S)\right\}_{j=1}^{J}$, (iii)the governments' default decisions $\{d_j(S)\}_{j=1}^{J}$ and the associated default sets $D_j(b_j \mid \{s_k\}_{k=1,k\neq j}^{J}, W)$, (iv) the representative investor's consumption $c^L(S)$, (v) the representative investor's holdings of emerging economies' bonds $\left\{\theta'_j(S)\right\}_{j=1}^{J}$, (vi) the representative investor's holdings of T-Bills $\theta^{TB\prime}(S)$, and (vii) the emerging economies' bond price functions $\left\{q_j(S, b'_j)\right\}_{j=1}^{J}$, such that:

(i) Taking as given the representative investor's policies and the bond price functions $\left\{q_j(S, b'_j)\right\}_{j=1}^{J}$, the emerging economies' consumption $\{c_j(S)\}_{j=1}^{J}$ satisfy the economies resource constraints. Additionally, the policy functions $\left\{b'_j(S)\right\}_{j=1}^{J}$, $\{d_j(S)\}_{j=1}^{J}$ and default sets $D_j(b_j \mid \{s_k\}_{k=1,k\neq j}^{J}, W)$ satisfy the optimization problem of the governments of the emerging economies.

(ii) Taking as given the governments' policies and the bond price functions $\left\{q_j(S, b'_j)\right\}_{j=1}^{J}$, the representative investor's consumption $c^L(s)$ satisfies the investor's budget constraint. Also, the representative investor's policy functions $\left\{\vartheta_j'(S)\right\}_{j=1}^J$ and $\vartheta^{TB\prime}(S)$ satisfy the optimization problem of the representative investor, and the law of motion of the investor's wealth.

(iii) Bond prices reflect the governments' probabilities of default and the risk premiums demanded by the representative international investor. These prices clear the market for all the emerging economies' bonds:

$$
\begin{aligned}
b_j{}'(S) &= -\vartheta_j{}'(S) \qquad \text{if } \; b_j{}'(S) < 0 &\quad (14a)\\
0 &= -\vartheta_j{}'(S) \qquad \text{if } \; b_j{}'(s) \geq 0. &\quad (14b)
\end{aligned}
$$

These equilibrium conditions in the bond markets imply that the representative investor, and the governments of each emerging economy agree on a financial contract $\left\{(b_j', q_j)\right\}_{j=1}^J$ that is optimal for both agents.

4 Contagion

Observing equation (13) it becomes evident that bond prices of the economy j depend on the income realizations of other emerging economies in the investors' portfolios, and the associated default/repayment decisions of the governments of such countries.

Hence, considering a crisis in some foreign emerging economy k as a shock that changes the expected default/repayment decisions of the government of the country, and therefore the default probability and the bond price of such emerging economy, a crisis in the emerging economy k has a wealth and a substitution effect over the optimal investor's portfolio allocation to other emerging economies.

Wealth Channel of Contagion First, the crisis in country k has a negative current or expected wealth effect. Because the investor's preferences exhibit DARA she would move away from risky emerging economies' assets towards safer assets; this effect corresponds to the channel refer in the introduction as the *wealth channel of contagion.* This channel of contagion operates when country k defaults in this period, reducing the current wealth of the investors and their tolerance of risk.

Proposition 3 *There is a wealth channel of contagion. Proposition 2 implies that if* *economy k which is in the investor's portfolio defaults in her debts, incentives to default for economy j which is also in the investor's portfolio increase.*

Proof. See appendix. ■

The intuition of Proposition 3 is straight forward: a default by some emerging economy in the investors' portfolio is equivalent to a negative wealth shock. Therefore, from Proposition 2 the probability of default for other economies in the investors' portfolio increase as a consequence of the default by economy k.

This result also implies that there is an expected negative wealth effect whenever the probability of default increases for some emerging country in the investors' portfolio. This wealth effect would tend to reduce the long-term benefits of maintaining access to international credit markets, and therefore would tend to increase the probability of default for the other countries in the investors' portfolio.

The Recomposition Channel of Contagion Second, the crisis in country k generates substitution between different risky emerging economy assets in the investor's portfolio. The substitution effect of the crisis corresponds to the channel refer in the introduction as the *portfolio recomposition channel of contagion*. This channel of contagion operates when the default probability of country k increases in this period reducing the expected wealth of the investor for the following period, making risky countries less attractive to the investor, and less risky ones more attractive to the investor. In this situation countries with weak fundamentals, that are reflected in high default probabilities, experience contagion, and the countries with solid fundamentals, that are reflected in low default probabilities, experience flight to quality. [13]

Previous literature in the subject of contagion in partial equilibrium model (see Kodres and Pritsker (1998)) have identified that the extend of the impact of the shock in one asset over another asset depends on the degree of correlation between the returns of those two assets. In the context of this model this implies that the impact of a shock in economy k over a particular economy (contagion or flight to quality) depends on the strength of the

[13] According to Kaminsky, Lyons, and Schmukler (2001) flight to quality occur when during the two first quarters after the Mexican crisis mutual fund flows to countries like Malaysia, Colombia, Poland and Czech Republic increased by more than 10%. Also during the two first quarters after the Thai crisis, mutual funds flows to countries like Venezuela, Slovak Republic and Sri Lanka increased by more than 5%. Finally, during the two first quarters after the Russian crisis, mutual funds flows to Mexico and Singapore increased by more than 5%.

positive correlation between the asset returns of economy k ,and asset returns of the other economies that share the investors with economy k.

Quantitatively, the model in here exhibits the following: If the positive correlation between q_k and q_j (for $j \neq k$) is low the positive substitution effect of the crisis in country k might dominate its negative wealth effect, and there is flight to quality. This is the case when investor's wealth is sufficiently large, or the investors are not too risk averse, or the other economies in the investor's portfolio have sound fundamentals.

In the other hand, when the positive correlation between q_k and q_j (for $j \neq k$) is large then the negative expected wealth effect dominates, and contagion due to the portfolio recomposition by the investors is observed. This is the case if either the wealth of the investors' is low, or the expected wealth shock is large because the exposure of the investor to the country with problems is large, or the investor's risk aversion is high.

Nevertheless the complexity of the model does not allows us to present an analytical proposition that establishes the unambiguous operation of the portfolio recomposition channel, and the possibility of flight to quality in this context. However, a much simpler version of the model that tries to maintain the most aspects of the model in here will illustrate the operation of the portfolio channel of contagion:

Example 5 *Three period model of contagion*

Lets consider three countries (A,B,C) that share a risk averse investor that lives for three periods, and has the following periodic utility: $u(c^L) = \log(c^L)$*. The investor owns an investing fund through which she lends to the countries. To simplify the portfolio allocation problem it is assumed that the investor has an initial amount of resources in that fund that can be used only to invest in emerging economies, and any resources left in the fund at the end of the third period will be used to pay for the fund's operating expenses.*

Every period the countries borrow from the fund a fix amount of X*. In any period, any country might default in its debts with the investor's fund. If any country defaults the investor's fund does not lend to that economy anymore. The investor's fund lends to the economies by buying the economies bonds at a price* q^i *for* $i = A, B, C$*, so the fund lends to each borrowing economy an amount* $q^i X$*, and the following period receives a repayment of* X *from each of the economies that do not default in their debts.*

When an economy pays back its debt to the fund, the investor receives from her fund a fix earning of fX *with* f *being a constant such that* $0 < h < 1$*; the investor can use this earnings at it pleases. Therefore in periods* $t = 2, 3$*, the investor receives earnings* $G_t = z_t hX$*, where*

z_t is the number of countries in period t that repay their loans. In addition to the portfolio earnings, in each period of her life the investor receives fix endowment of $\overline{U}$.

The investor maximizes her lifetime utility from consumption and has access to international credit markets from which she can borrow or lend by issuing risk free bonds at a price q^f. To simplify the optimization problem of the investor, it is assumed that at the beginning of the initial period $t = 1$ she believes that she only will be alive during two periods; only until the beginning of period $t = 2$ the investor realizes that she is going to be around for an additional period.[14].

In the other side of the market the emerging economies $i = A, B, C$ ha a production technology given by $Y^i = \alpha \left\{ q^i X \right\}$ if the country can borrow from the investor's fund, and $Y^i = \phi$ when the country is not able to borrow from the investor's fund. Every period t after the country has borrowed from the investor's fund and have produced its good, the country sets apart the repayment for the investor's fund X. However, with an exogenous probability δ_t^i this resources are destroyed by social upheaval at the beginning of period $t + 1$ (that is the period when the payment of the debt is due).[15] *It is assumed that in period $t = 1$, all countries are identical, and therefore they have the same productivity α, and default probability δ_1.*

Regarding the the pricing of the bonds of the economies q^i it is assumed as in quantitative model of the paper: using the utility function of the investor. Therefore the profit function of the investor's fund is not described in here (or in other words, it is assumed that the profit function of the investor's fund is identical to the investor's utility function). Another important assumption is that the pricing of the bonds takes in account the information of the investor regarding the duration of her life, therefore q_1^i only depends on what occurs in periods $t = 1, 2$, and q_2^i only depends (by construction) on what occurs in periods $t = 2, 3$.

Investors's optimization problem *The budget constraints of the investor are given by the following equations:*

[14] In this way when the investor borrows or saves to smooth her consumption she only looks at the current and future period (i.e., in period $t = 1$ she borrows to smooth her consumption over periods $t = 1$ and $t = 2$, and does not consider period $t = 3$ at that moment)

[15] α corresponds to a productivity parameter; it is assumed that α is sufficiently high such that whenever the country repays, its consumption is larger than 0.

$$
\begin{aligned}
c_1^L &= \overline{U} - S_1 q^f \\
c_2^{*L} &= \overline{U} + S_1 + G_2 \\
c_2^L &= \overline{U} + S_1 + G_2 - S_2 q^f \\
c_3^L &= \overline{U} + S_2 + G_3
\end{aligned}
$$

where c_n^L is the actual investor's consumption in the period $t = n$; c_2^{*L} is the consumption that the investor thinks she will have in period $t = 2$ at $t = 1$. S_n is the saving of the lender in period $t = n$; G_n are the earnings coming from the lending activities of the investor's fund that the investor receives in $t = n$.

Taking q^f and G_n as given, the investor chooses S_1 to solve the following maximization problem in period $t = 1$:

$$
\underset{S_1}{Max}\ u\left(c_1^L\right) + \beta E_1 u\left(c_2^{*L}\right).
$$

In the same way she chooses S_2 to solve:

$$
\underset{S_2}{Max}\ u\left(c_2^L\right) + \beta E_2 u\left(c_3^L\right).
$$

Therefore the first order conditions for S_1 and S_2 are given by:

$$
S_1 \quad : \qquad q^f u_c(c_1^L) = \beta E_1 u_c(c_2^{*L}). \tag{15}
$$

$$
S_2 \quad : \qquad q^f u_c(c_2^L) = \beta E_2 u_c(c_3^L). \tag{16}
$$

Countries' problem *In the periods in which country i is borrowing from the investor's fund its consumption is given by the following budget constraint:*

$$
c^i = Y^i - X = (q^i \alpha - 1)X.
$$

In the periods in which the country cannot borrow from the lender its budget constraint implies:

$$
c^i = Y^i = \phi.
$$

Countries' equilibrium bond prices *The assumptions about the pricing of the bonds of the emerging economies imply that q^i must be such that it covers the expected default losses, and a risk premium that a risk averse lender has to receive in order to invest in the risky countries.*

Given the methodology for the pricing of the bonds q_1^i and q_2^i for any country i are given by

$$q_1^i u_c(c_1^L) = \beta E u_c(c_2^{*L}) d_2^i. \tag{17}$$
$$q_2^i u_c(c_2^L) = \beta E u_c(c_3^L) d_3^i. \tag{18}$$

where $u_c(\cdot)$ is the marginal utility of consumption of the investor, and d_t is an indicator variable that tells if the economy i repays its debt in periods $t = 2, 3$. If economy i pays $d_t^i = 1$, otherwise $d_t^i = 0$.

Solution *We solve the model using the following parametrization of the model: $u(c_t^L) = log(c_t^L)$, $\beta = 0.95$, $q^f = \beta$, $\alpha = 2/\beta$, $\phi = 0.25$, $\delta_1^i = 0.1$ for $i = A, B, C$, $\delta_2^i = 0.1$ for $i = A, B, C$, $hX = 50$, and $\overline{U} = 100$.*

Given these parameters, we obtain the following results for the first period :

Table 1: **First Period: Simple Model of Contagion**

S_1	q_1^i	c_1^i
−66.211	0.8178	$0.7217X$

It is worthwhile to notice that if the lender was a risk neutral lender, the price of the bonds would be given by:

$$q_1^{i,RN} = (1 - \delta_1^i) * q^f \tag{19}$$

In this case the results for the first period would have been:

Table 2: **First Period, Risk Neutral Lender**

$q_1^{i,RN}$	$c_1^{i,RN}$
0.855	$0.8X$

Simple Model: Wealth Channel of Contagion *This subsection focus on the effect that a default by other country has in the wealth of the lender, and how that affects the prices of the bonds of the economies that have not defaulted.*

Period 2

Taken as given the savings decision of the lender in the first period, S_1, there are four different scenarios for the lender's maximization in $t = 2$:

Case 1*: all countries payed their debt from period $t = 1$. Case* 2*: two countries payed their debt from period $t = 1$. Case* 3*: only one country payed its debt from period $t = 1$. Case* 4*: no country payed its debt from period $t = 1$.*

Table 3: **Second Period: Wealth Channel of Contagion**

Case	S_2	q_2^i	c_2^i
Case 1	−24.219	0.8294	$0.7462X$
Case 2	−26.953	0.8195	$0.7254X$
Case 3	−29.883	0.7985	$0.6810X$
Case 4	−33.936	0.0000	ϕ

Case 1 *is a situation where the lender did not suffer any losses by lending to the risky countries. Cases* 2,3 *and* 4 *illustrate situations when the lender receives at $t = 2$ low earnings G_2 because some, or all countries have defaulted.*

Comparing case 1 *to the other cases we observe that when some country in the portfolio of the lender defaults the countries that do not default have nonetheless to accept a lower price for their bonds. This in turn reduces these economies consumption, and all economies end up having a lower level of consumption than when no country defaults. Therefore there is contagion of the crisis from the country or countries that default to the other countries in the lender's portfolio.*

For example, if economy A pays back in $t = 2$ but some or all other countries do not, then instead of having $c_2^A = 0.7462X$ economy A gets a consumption that is 2.79% *lower if either B or C defaults, and that is* 8.74% *lower if both default.*

This situation of the lender getting lower earnings when the countries default in period $t = 2$ is analogous to the situation in the model in this paper when the investor's has a lower W_t as consequence of a default by some economy at period t. The results of in here illustrate the **Wealth Channel of Contagion** *that is also in the fully-fledge model of the paper.*

Simple Model: Portfolio Recomposition Channel *The result in this subsection focus on the effect that an increase in the probability of default of a country has in the other countries. We assume that the probabilities of default of the first period are as before, but now for a country C the probability of default increases from* $\delta_2^C = 0.1$ *to* $\delta_2^C = 0.2$*. The probability of default of the other two countries remains the same as before. Since at the beginning of period* $t = 2$ *the lender does not know that she will be alive for three periods, the results of the first period are unchanged.*

Period 2

Again there are several different scenarios for the lender's maximization in the second period:

Case 1: all countries payed their debt from period $t = 1$*. Case* $2A$*: two countries payed their debt from period* $t = 1$*; both countries have a default probability in* $t = 2$ *of* 0.1*. Case* $2B$*: two countries payed their debt from period* $t = 1$*; one of the countries have a default probability in* $t = 2$ *of* 0.1*, and the other has a default probability in* $t = 2$ *of* 0.2*. Case* $3A$*: only one country payed its debt from period* $t = 1$*; this country has a default probability in* $t = 2$ *of* 0.1*. Case* $3B$*: only one country payed its debt from period* $t = 1$*; this country has a default probability in* $t = 2$ *of* 0.2*. Case* 4*: no country payed its debt from period* $t = 1$*.*

Table 4: **Second Period: Portfolio Recomposition Channel of Contagion**

Case	S_2	q_2^A	c_2^A	q_2^B	c_2^B	q_2^C	c_2^C
Case 1	-21.094	0.8285	$0.7441X$	0.8285	$0.7441X$	0.7160	$0.5074X$
Case 2A	-26.953	0.8195	$0.7254X$	0.8195	$0.7254X$	0.0000	ϕ
Case 2B	-23.633	0.8174	$0.7208X$	0.0000	ϕ	0.6999	$0.4735X$
Case 3A	-29.883	0.7985	$0.6810X$	0.0000	ϕ	0.0000	ϕ
Case 3B	-26.367	0.0000	ϕ	0.0000	ϕ	0.6691	$0.4085X$
Case 4	-33.936	0.0000	ϕ	0.0000	ϕ	0.0000	ϕ

We observed that there is not need for a country in the portfolio of the investor to default to observed contagion; just the increase in the default probability of a country in the investor's portfolio reduces the bond prices of the other countries, and their consumption.

For example, comparing the results in case 1 in this subsection with the results in case 1 of the previous subsection, where all countries in the investor's portfolio had a default probability of 0.1*, we observe that when the probability of one country in the investor's portfolio increases from* 0.1 *to* 0.2 *the other countries A and B consume* 0.28% *less than if the probability of C had remained constant.*

The contraction in the consumption of a country A in response to the change in the probability of default of country C is even larger when the lender is suffering losses from a default by the other country in her portfolio: comparing cases $2A$ with $2B$ we find that country A has a consumption that is 0.63% *lower when the probability of country C increases from* 0.1 *to* 0.2 *if B defaults in its debt in $t = 1$ than when B pays its debts. This contraction in the consumption of A is almost* 3 *times larger than the contraction in such consumption that occurs when no country defaults in $t = 2$.*

This situation of a country suffering a reduction in its consumption when another country's default probability increases is analogous to the situation in the quantitative model of the paper of having a country that shares the investor facing a negative income shock that increases its default probability. As in this example, in the fully-fledge model of the paper the shock to the other country in the investor's portfolio results in a reallocation of the investor's portfolio away from risky investments. The results of this subsection illustrate the **Portfolio Recomposition Channel of Contagion** *discussed in the paper.*

Simple Model: Flight to quality *In this subsection the focus is on the effect that the increase in the probability of default of a country has in the other countries depending on the riskiness of those other countries. In this exercise a country with a medium probability of default (*0.1*) receives an exogenous shock that increases its default probability (to* 0.25*). If the investor is aware that when the consumption of the countries falls their probability of default rises (as it is in the fully-fledged model in the paper), then a country with relative low risk might experience capital inflows after the initial shock to the probability of default of the other country. This phenomena is identified as* flight to quality *in the literature of financial flows. We assume that $t = 1$ is as in the previous subsections.*

Period 2

In $t = 2$ country A has low default probability (0.02*); country B has initially the same probability as before (*0.1*), and then it receives a shock that increases sharply its probability of default (to* 0.25*); country C has a high probability of default (*0.2*). For briefness we are going to focus only in the case in which all three countries repay their debts from period $t = 1$.*

As before, if the probability of default of countries A and C does not change when the probability of default of country B increased, all countries suffer contagion by the shock to country B. Nevertheless it is important to notice that the shock hits much more stronger the riskier country C: the shock reduces consumption in country A by 0.31705% *while*

Table 5: **Second Period: Flight to Quality-Exogenous Default Probabilities**

δ_2^B	S_2	q_2^A	c_2^A	q_2^B	c_2^B	q_2^C	c_2^C
0.1	−23.828	0.9259	$0.9492X$	0.8298	$0.7469X$	0.7176	$0.5107X$
0.25	−19.141	0.9244	$0.9462X$	0.6618	$0.3932X$	0.7152	$0.5058X$

consumption in country C falls by 0.96876%*, three times more.*

In what follows we are going to modify the example to have, as in the fully-fledged model in the paper, an effect in the default probability of the countries that suffer contagion.

In order to look at flight to quality in the example, the assumption is that after the initial exogenous shock to the default probability of country B the lender takes in account the changes in the default probability of the countries that suffer contagion, and makes sequential reallocations of her portfolio for a given number of times. The results presented in here have the lender making 5 *rounds of sequential reallocations of her portfolio; the number of rounds of reallocations have important implications for the results of the simple model, these implications will be discuss briefly at the end of this part.*

It will be assumed that the probability of default coming from contagion after each round of reallocation of the lender's portfolio follows this functional form: [16]

$$\delta_2^{i,g} = \delta_2^{i,g-1} \left(\frac{c_2^{i,g-2}}{c_2^{i,g-1}} \right)^{\frac{c_2^{i,g-2}}{c_2^{i,g-1}}}$$

where $i = A, C$ and g corresponds to the reallocation round.

Comparing the results in this table to the results in the previous table with exogenous probability of default for countries A and C, we observe flight to quality: the bond prices of economy A increase in 0.022% *instead of falling in* 0.16%*, and its consumption increases in* 0.021% *instead of falling in* 0.316%*; in the other hand the prices of economy C fall in* 1.1% *instead of falling in* 0.335%*, and its consumption falls in* 3.25% *instead of falling in* 0.959%*. The probability of default of the economy A falls from* 2%*, to* 1.99%*, while the probability of default of economy C increases from* 20% *to* 20.62%*.*

Because now country A experiences positive contagion this exercise in the simple model

[16]The functional form of the default probability it is important to achieve the results in here, in particular it is necessary that $\frac{\partial \delta^i}{\partial c^i} < 0$, and $\frac{\partial^2 \delta^i}{\partial {c^i}^2} < 0$.

Table 6: **Second Period: Flight to Quality-Endogenous Default Probabilities**

Round	S_2	δ_2^A %	q_2^A	c_2^A	q_2^B	c_2^B	δ_2^A %	q_2^C	c_2^C
1	-19.141	2.0064	0.9249	$0.9472X$	0.6621	$0.3940X$	20.1956	0.7135	$0.5021X$
2	-19.141	2.0043	0.9253	$0.9481X$	0.6624	$0.3945X$	20.3455	0.7122	$0.4993X$
3	-19.141	2.0024	0.9257	$0.9487X$	0.6626	$0.3949X$	20.4603	0.7111	$0.4971X$
4	-19.141	2.0011	0.9259	$0.9493X$	0.6628	$0.3953X$	20.5512	0.7103	$0.4954X$
5	-19.141	1.9999	0.9261	$0.9497X$	0.6629	$0.3955X$	20.6220	0.7097	$0.4941X$

illustrates the phenomena described referred in the discussion of the fully-fledged quantitative model as **flight to quality**.

The observation of flight to quality in the simple model depends on the number of rounds of sequential reallocations that the lender does. If the reallocation process continues for a while the effect of flight to quality can disappear if the endogenous increase in the riskiness of C dominates, in terms of the overall riskiness of the portfolio, the diminishing in the riskiness of A.

For example, if the rounds of reallocations of the lender's portfolio in this exercise are 8 *instead of* 5*, then after the* 8*th reallocation δ_2^C has increased in* 3.61% *while δ_2^A has decreased in only* 0.08%. *At this point the increase in the default probability of C is too large, and triggers capital outflows from all countries, at the same time the lender reduces her borrowing from international credits markets.*

Therefore an additional conclusion that can be draw from this result is that, taken as given the probabilities of default of the other countries in the lender's portfolio, there are some threshold values for the probability of default of a country A that might determined if this country suffers contagion or gets flight to quality: For example, in this simple model if $\delta_2^B = 0.25$ and δ_2^C increases from 0.2068 to 0.2072 we have that if $\delta_2^A \leq 0.01972$ then country A receives capital inflows, but if $\delta_2^A > 0.01972$ country A gets capital outflows.

This result of the simple model illustrates the point in the discussion of the critical value of the default probabilities to determined the effects of a recomposition of the portfolio of the investors on an specific country's bond prices.■

From the results in the simple model, which shares the main elements of the fully-fledged model in the paper (i.e., bond pricing for sovereign risk bonds based on the utility function of risk averse investors with DARA preferences and where the risk premium on the bonds is endogenous and when the default probability is also endogenous depends negatively of the consumption of the economy), it is possible to establish the following general results that apply also to the fully-fledged model because they are based on shared elements of the models:

Proposition 4 *There is a recomposition channel of contagion. Whenever the probability of default of some country k increases there is a recomposition of the investor's portfolio away from risky countries.*

Proposition 5 *For economies with strong fundamentals the effect of the crisis in some foreign economy k would be "flight to quality".*

In the simple model this recomposition implies a smaller price for the bonds of risky economies, and larger bond prices for safe economies, which in equilibrium implies that risky countries receive smaller capital inflows and therefore have lower levels of consumption, and safer countries receive larger capital inflows and have higher levels of consumption. In the fully-fledged model of the paper this portfolio recomposition implies that the investor moves away from the assets of emerging economies with weak fundamentals towards other emerging economies or T-Bills.

Also from the simple example we can derive the following result that applies to both models:

Corollary 6 *Given the riskiness of the other countries in the investor's portfolio, there is a critical value for the riskiness of the domestic economy that determines if the economy experiences flight to quality or negative contagion.*

This implies that in the determination of the effect of the crisis of a country with financial links to a domestic economy the strength and direction of this effect will be determined by the fundamentals of the domestic economy: sufficiently strong fundamentals imply experiencing 'flight to quality" and weak fundamental or not sufficiently strong fundamentals imply experiencing contagion.

5 Quantitative Analysis

As it is well know during 2001 Argentina faced one of the worst economic crisis of its history. This crisis forced the country to default in $US100$ billion external government debt (nearly that corresponded to nearly 37% of GDP)by the end of 2001, and had strong real effects during 2001 that extended into 2002: according to estimates of the IMF during 2001 Argentina GDP fell by 4.4% and during 2002 Argentina's GDP fell by and additional 10.9%.

Uruguay, in other hand was facing some economic problems since 1998 that were aggravated in 2001 by the outburst of cow foot-and-mouth disease that affected very negatively its exports. Additionally, the Argentinean crisis weakened confidence prompting caution in consumers and investors and cause the real demand to fall at the same time that caused an exchange rate depreciation of the Uruguayan peso that generated a significative increase in the public debt to GDP ratio (this ratio went from .. to 52%). According to the estimates of the IMF during 2001 Uruguay's GDP fell by 3.5%, and during 2002 Uruguay's GDP fell by and additional 7.1%. The GDP's fall in 2002 was due mainly to problems in Uruguay's financial sector which has strong financial links to Argentina: in early 2002 following the Argentina's default Uruguay' financial sector experienced large dollar deposit outflows (these outflows exceeded 100 million per day in the month of July 2002), and as it faced a rapid decline in its international reserves (Uruguay's international reserves fell from 3 billion dollars at the end of 2001 to 650 million by August 2002). During 2002 Uruguay's debt was downgraded by investment rating agencies signaling the credit risk involved in Uruguay's external debt. [17].

The simulation of the model in this paper analyzes the Argentinean default of 2001 and its implications in Argentina's neighbor Uruguay. [18]

[17]Uruguay's deposit outflows were initially confined to non-residents but they accelerated in March 2002 spreading to resident depositors as the result of problems of two local banks and the Argentina's crisis. In early August the central Bank suspended operations of four private domestic banks, and reprogrammed foreign currency time deposits in public banks by up to three years, and established a fund designed to fully back existing foreign currency sight deposits at domestic banks which helped to stop the deposit outflows for the remainder of the year. A new round of deposits outflows took place in January 2003, gross official reserves fell to US540 million by end-February 2003 (equivalent to only half of the domestic reserve liabilities of the central bank)

[18]Uruguay was the most affected country, by the spillovers of the Argentinean crisis. While capital flows to Latin America dipped sharply in the summer and fall of 2001 in response to the crisis in Argentina, by the end of the year they were recovering despite the expectation of Argentina's default. Contagion in the bond market had been significant until October 2001 with spreads throughout Latin America rising in step of the deterioration of Argentine credit risk. But after October the spreads in other Latin America bonds declined markedly and by March 2002 they had reached their lowest levels since April 1998 and bond issuance remained healthy. Uruguay, with particulary close financial links to Argentina suffered strong pressures on its currency. Other Latin American countries were not as affected as Uruguay mostly because trade and financial links between Argentina and most of its neighbors remain relatively limited, only in Bolivia, Brazil, Chile, Paraguay and Uruguay does Argentina accounts for more than 10% of total trade. [19] In the other hand, and in line with the previously discussed theoretical results of this article that establish that contagion occurs mainly in those countries with weak fundamentals, sound macroeconomic management in most of the region increased confidence of international investors. Finally, an environment of ample global liquidity

Table 7: Business Cycle for Argentina and Uruguay - period for which data are available

Variable	mean(x)	std(x)	corr(x,y^{AR})	corr(x,r^{AR})	corr(x,y^{UR})	corr(x,r^{UR})	corr(x,y^{US})
Spread Argentina ($(r-r_f)^{AR}$)	12.67	5.42	-0.60		-0.44	0.18	-0.11
Trade Balance Argentina (TB^{AR})	0.58	2.93	-0.59	0.38	-0.47	0.05	0.08
Output Argentina (y^{AR})	100.00	1.91			0.63	-0.24	-0.02
Consumption Argentina (c^{AR})	78.32	7.88	0.93	-0.78	0.71	-0.63	0.60
Spread Uruguay ($(r-r_f)^{UR}$)	8.53	1.33			-0.30		-0.25
Trade Balance Uruguay (TB^{UR})	-10.67	4.27	-0.26	0.16	-0.48	0.09	-0.17
Output Uruguay (y^{UR})	100.00	1.64					0.49
Consumption Uruguay (c^{UR})	68.32	6.83	0.55	-0.31	0.86	-0.13	0.42

Table 8: Business Cycle for Argentina and Uruguay - the year of the crisis

Variable	mean(x)	std(x)	corr(x,y^{AR})	corr(x,r^{AR})	corr(x,y^{UR})	corr(x,r^{UR})	corr(x,y^{US})
Spread Argentina ($(r-r_f)^{AR}$)	22.26	13.59	-0.96		-0.80	0.52	-0.71
Trade Balance Argentina (TB^{AR})	2.13	2.64	-0.85	0.89	-0.94	0.58	-0.79
Output Argentina (y^{AR})	95.6	3.71			0.84	-0.70	0.86
Consumption Argentina (c^{AR})	76.99	10.38	0.98	-0.99	0.83	-0.60	0.77
Spread Uruguay ($(r-r_f)^{UR}$)	9.53	1.45			-0.81		-0.88
Trade Balance Uruguay (TB^{UR})	-9.30	5.62	-0.24	0.30	0.26	-0.12	0.19
Output Uruguay (y^{UR})	96.50	5.93					0.91
Consumption Uruguay (c^{UR})	100.46	6.59	0.88	-0.77	0.77	-0.63	0.91

Tables 7 and 8 describe the relevant business cycle features for the periods under study: Table 7 describes the statistics for the entire period for which data is available while Table 8 describes the statistics for the year of the crisis. For the Argentinean output, consumption and trade balance the source of the data is the IFS. For the interest rate of Argentina the source is Neumeyer and Perri (2005). For Uruguay the series for output, consumption and trade balance are constructed using the Uruguayan Central Bank quarterly and annual data on indexes and volume for these variables. Also, for Uruguay's interest rate the source is the Uruguayan Central Bank, and this rate corresponds to the domestic interest rate in loans, which clearly is not the interest rate in international loans, but should be positively correlated with such external interest rate (there is not EMBI for Uruguay and while it would be possible to calculate an implicit interest rate from Uruguay's debt service data, such data is available only annually).

The data for the business cycle statistics includes the period 1983:Q1-2001:Q4 for the all the Argentinean series except the consumption series which is only available for the period 1993:Q1-2001:Q4; For the Uruguayan series the period for which the data are available corresponds to 1988:Q1-2001:Q4 for output, consumption, and trade balance, and to 1980:Q1-2001:Q4 for the interest rate. Therefore, the business cycle statistics for each vari-

favored large and relatively liquid markets such as Mexico and Brazil.

able correspond to the initial moment in which each of them is available to the fourth quarter of 2001. The correlations are taken for the common periods in which any pair of variables are available. Output and consumption for Argentina and Uruguay are seasonally adjusted and are in logs and filtered with the H-P filter. Argentina's and Uruguay's trade balances are reported as a percentage of their respective output. The interest spread is defined as the difference between the Argentinean and the Uruguayan interest rate and the yield of a 3 month U.S. T-Bill.

From the tables is possible to observe that both Argentina's and Uruguay's interest rates and trade balances are counter-cyclical which are well known facts for the case of emerging economies. More importantly for our analysis, the interest rates and trade balances of these two countries are also negatively correlated with their neighbor's GDP, and these negative correlations are larger during the period of the Argentinean crisis: the correlation between Argentina's GDP and Uruguay's interest rate for the whole period of study is -0.24 and becomes -0.80 during the year of the crisis; the correlation between Uruguay's GDP and Argentina's interest rate for the whole period of study is -0.44 and jumps to -0.80 during the period of the Argentinean crisis. The behavior of the correlations of the countries trade balances and their neighbor's GDP follows a similar pattern to the one observe for the correlations of the interest rates and the GDP's of the neighbor country.

For the analysis in here is also important to notice that Argentina's and Uruguay's interest rates are positively correlated (0.18), and this correlation is much larger during the period of the Argentinean crisis (0.52). Finally, the consumptions in these two countries are positively correlated with the neighbor country's GDP and the correlations are larger during the year of the Argentina's crisis: the correlation between Argentina's consumption and Uruguay's GDP is 0.71 and 0.83 for the whole period and the crisis period respectively; the correlation between Uruguay's consumption and Argentina's GDP is 0.55 and 0.88 for the whole period and the crisis period respectively .

Given the assumption of the model of identical economies that only differ in the realizations of their endowments, and in order to facilitate comparison with the previous literature in the subject, the parameters consider for the simulation are chosen to replicate some features of the Argentinean economy, and are taken from the calibration for this economy in Arellano (2008). The parameters related to international investors are taken from Lizarazo(2011).

Table 9 shows the parameters of the numerical analysis of the model. The mean income of the emerging economy is normalized to 1. The coefficient of risk aversion of the economy is 2, a standard value considered in the business cycle literature. The free interest rate

Table 9: Contagion: Parameter Values

Parameter	Value
Emerging Economy's Mean Income $E[y]$	1
Std. Dev. Emerging Economy's Income $std[y]$	0.025
Autocorr. Emerging Economy's Income Process	0.945
Emerging Economy's Discount Factor β	0.953
Emerging Economy's Risk Aversion γ	2
Probability of re-entry τ	0.282
Critical level of output for asymmetrical output cost	$\hat{y} = 0.969E(y)$
Representative investor's Income X	0.01
Representative Investor's Discount Factor β^L	0.98
Representative investor's Risk Aversion γ^L	2
Risk Free Interest Rate $r^f = \frac{1}{q^f}$	0.017

is set to 1.7%, to match for the period under study the quarterly US interest rate of a bond with a maturity of 5 years. The GDP is assumed to follow a log-normal AR(1) process $log(y_t) = \rho log(y_{t-1}) + \varepsilon^y$ with $E[\varepsilon^y] = 0$ and $E[\varepsilon^{y2}] = \sigma_y^2$. The values estimated by Arellano(2008) for the Argentinean economy are $\rho = 0.945$ and $\sigma_y = 0.025$, the shock is discretized into a Markov chain using the quadrature based procedure (Hussey and Tauchen (1991)). Following a default there is an asymmetrical function for the output loss that is as follows:

$$\phi(y) = \left\{ \begin{array}{cc} \widehat{y} & \text{if } y > \widehat{y} \\ y & \text{if } y \leq \widehat{y} \end{array} \right\} \tag{20}$$

with $\widehat{y} = 0.969E(y)$ which in Arellano (2008) targets a value of 5.53% for the average debt service to GDP ratio. The probability of re-entry to credit markets after defaulting is set at 0.282, which is consistent with the empirical evidence regarding the exclusion from credit markets of defaulting countries (see Gelos et al. (2002)), and that in Arellano (2008) targets a volatility of 1.75 for the trade balance. The discount factor is set a 0.953 which in Arellano (2008) targets a annual default probability of 3%.

The parameters for the international investors are as follows: the representative investor's discount factor is set to 0.98. As in Lizarazo (2011) if there were no uncertainty, the discount factor of the investors would pin-down the international risk free interest rate (i.e., $\frac{\beta^L}{q^f} = 1$); however with uncertainty in order to have a well defined distribution for the investor's assets is necessary to have a value of the discount factor such that $\frac{\beta^L}{q^f} < 1$. The value of $\beta^L = 0.98$ is the highest value in the range commonly used in business cycle studies of industrialized countries such that for an international interest rate of 1.7% the asset distribution of the investors is well defined.

Following Lizarazo (2011) the representative investor's coefficient of risk aversion is set at 2 and the criteria to choose this parameter is to generate a mean spread for model that is as close as possible to the mean spread in Argentina for the period of study, which corresponds to 12.67%.

The representative investor receives a deterministic income of $X = 1\%$ of the emerging economy's mean income in each period. As in Lizarazo (2011) this parameter is included to preclude the investors from not investing in the emerging economy in order to avoid a negative consumption level in the case of default by the government of the economy. Therefore, the strategy for choosing X is to give it as little importance as possible by choosing a value that is close to 0 and that still allows for interior solutions regarding the investors' investments in the emerging economy's bonds. Overall, the numerical analysis of the model shows that as long as X is not too large (i.e. $X < 100\%$ of the emerging economy's average income) the results of the model are not very sensitive to the value of X.

5.1 Simulations

By considering the fundamentals of countries that share investors, the simulations presented here aimed to replicate the following observed dynamics of sovereign yield spreads, and capital flows to emerging economies: i) the sovereign risk premium is high during recessions, or when the economy is highly indebted; ii) default is observed when the fundamentals of the economy deteriorate, iii) in periods previous to default the economy experiences capital outflows and collapses in consumption, iv) capital flows and domestic interest rates across emerging economies are positively correlated, and iv) default is more likely to be observed when the fundamentals of other emerging economies deteriorate.

The model is simulated for two economies (Argentina and Uruguay) that are labeled as A and U respectively. For each economy the endowment shock is discretized into a 5 state Markov chain and the asset positon of the economy is approximated by a 75 points grid. For the investor's her wealth level is approximated using a 10 grid, over which the solution to the investor's problem is linearly interpolated. [20] The business cycles statistics of the

[20] In both the discretization of the endowment realization and the asset position of the emerging economies there are important differences with the calibration in Arellano (2008): in that article the endowment realization has 21 states and the economy's asset position state variable has 200 states. For a model like the one in here, with more two economies such dimension in the economies' state variables is too

model are derived as follows: Each time the model is simulated for $20,000$ periods. From this $20,000$ periods, sub-samples that have economy A staying in the credit market for 74 periods before going into a default are taken to compute the business cycles statistics of the two economies. This process is repeated 5000 times, and the cycle statistics are the average of the statistics derived from each of these repetitions.

The business cycle statistics of the data are compared with the statistics generated by the model. The results of the simulations are shown in Table 10 for the whole period under study and in Table 11 for the year of the crisis. In these tables the results of the model of contagion are compared with the results of a simulation of the model with risk neutral investors that has the same number of endowment shocks and the economies' asset position than the model of the contagion.

Table 10 and Table 11 show that in general terms the contagion model fits the business cycle statistics of Argentina and Uruguay relatively better than the model without financial links (i.e, risk neutral investors). In both models a default occurs when economy's A GDP is in average 5.5% below its trend. In the contagion model, the effects of the default of country's A in country U's business cycle statistics occur even when in average country U's income is neither under or below its trend.

In the data, the spreads of Argentina are 12.67% for the whole period, and 22.26% during the crisis period (i.e., the year previous to a default episode). The contagion model generates a spread for the overall period of 5.7% and 12.7% for the crisis period, while the model without financial links only predicts a spread of 4.5% for the whole period and 4.9% for the crisis period. Unfortunately, regarding the volatility of the spreads both models (the contagion and the one without financial links) under-predict this volatility and they do by nearly the same magnitude.

The better matching of the spreads by the contagion model is not the result of a higher probability of default predicted by this model than by the model without final links since both models predict a annual probability of default of 3.8% . Also, the higher spreads do not imply a contraction in the mean debt level that the contagion model supports in comparison to the model without financial links, both models have an unconditional mean debt level for the whole period of 15.9%, and for both models the unconditional mean debt

large, since for the case of two countries, and taking in account the investor's wealth level it would imply $(21 \times 200) \times (21 \times 200) \times 10 = 176'400,000$ possible states for the whole model compare to $21 \times 200 = 4200$ states in Arellano (2008). The version of the model in here has $(5 \times 750) \times (5 \times 750) \times 10 = 1'406,250$ possible states, still a much larger dimension than the model with only one economy and risk neutral investors.

Table 10: Contagion: Business Cycle Statistics for 74 periods before the default of country A

Business Cycles Statistics for the Contagion Model and the Model with Risk Neutral Investors											
	Contagion Model with 2 Countries							Model with Risk Neutral Investors			
	Mean(x)	std(x)	corr(x,y^A)	corr(x,r^A)	corr(x,y^U)	corr(x,r^U)	corr(x,W)	Mean(x)	std(x)	corr(x,y^A)	corr(x,r^A)
Spread A	5.70	1.11	-0.06	–	-0.03	0.32	-0.34	4.5	0.6	-0.13	–
Trade Balance A	0.4	0.77	-0.16	0.38	-0.01	0.05	0.10	0.4	0.76	-0.15	-0.10
Output A	–	1.94	–	–	0	–	0.35	–	1.94	–	–
Consumption A	–	2.20	0.97	-0.04	0.01	-0.08	0.31	–	2.11	0.90	-0.14
	Default Probability = 1.16 Conditional Default Probability = 2.3							Default Probability = 1.16 Conditional Default Probability = 1.16			
	Mean Debt Level= -15.87 Conditional Mean Debt Level = -11.86							Mean Debt Level= -15.89 Conditional Mean Debt Level=-15.89			

Table 11: Contagion: Business Cycle Statistics for 4 periods before the default of country A

Business Cycles Statistics for the Contagion Model and the Model with Risk Neutral Investors											
	Contagion Model with 2 Countries							Model with Risk Neutral Investors			
	Mean(x)	std(x)	corr(x,y^A)	corr(x,r^A)	corr(x,y^U)	corr(x,r^U)	corr(x,W)	Mean(x)	std(x)	corr(x,y^A)	corr(x,r^A)
Spread A	12.40	0.81	-0.91	–	-0.80	0.88	0.89	4.9	1.0	-0.20	–
Trade Balance A	0.60	0.78	0.78	0.89	0.30	0.58	0.79	0.60	0.77	0.60	-0.22
Output A	–	5.19	–	–	0	–	0.85	–	5.19	–	–
Consumption A	–	5.03	0.99	-0.90	0.29	-0.30	0.84	–	4.62	0.68	-0.25
	Mean Debt Level = -20.14 Conditional Mean Debt Level=-12.20							Mean Debt Level = -20.17 Conditional Mean Debt Level=-15.89			

Table 12: Debt Securities Issue Abroad 1993(Q3)-2004(Q3)

Debt Securities Issued Abroad Correlations								
	Argentina	Brazil	Mexico	Russia	Philippines	Colombia	V	enezuela
Argentina	1	0.98	0.95	0.94	0.95	0.98	0	.88
Brazil		1	0.93	0.91	0.98	0.99	0	.91
Mexico			1	0.90	0.88	0.90	0	.81
Russia				1	0.91	0.91	0	.90
Philippines					1	0.98	0	.94
Colombia						1	0	.90
Venezuela							1	

Joint BIS-IMF-World Bank Statistics on External Debt

level for the period previous to the default episode increases to 20.1%.[21]

It is also noteworthy noticing that the probability of country U to default conditional that there is going to be a default episode by country A in the sample is 1.24% larger per year. This result of the model is consistent with the observed downgrading of the Uruguayan external debt by international credit rating agencies that occurred in light of the events of the Argentinean crisis. Also, for country U the mean debt level conditional to the fact that there is going to be a default in the sample is 11.9% for the whole period, and 12.2% for the year of the crisis, showing that what is going on with country A has important effects on country U's access to credit markets. [22] This effect of a country's fundamentals in other countries access to credit markets can see in Table 12 that shows a very high and positive correlation for the debt securities issued abroad for the group of emerging countries that include Argentina, Brazil, Mexico, Russia, Philippines, Colombia, and Venezuela during the period (1993:Q3-2004:Q3).

The contagion model reproduces as well as the model without financial links the counter-cyclical behavior of the spreads and the trade balances, and the pro-cyclical behavior of consumption for the whole period, and for some of these correlations does an even better job than the model without financial links in explaining them during the periods of crisis . Additionally, the contagion model is able to explain the correlations between the economies fundamentals and the wealth of the investors (proxy by the US's GDP) and the correlations

[21] The larger probability of default in this model as well as the larger level of debt supported at equilibrium by the models in comparison to the results in Arellano (2008) for a very similar calibration of the economy might be explain by the results in Hatchondo and Martinez (2006) that suggest that the results of endogenous sovereign risk models are somewhat sensitive to the solution method employed for the model as well as the dimension of the grid used to expand the asset position of the emerging economies

[22] The larger mean debt level that is observed during the periods of crisis can be explained by the fact that in order to smooth consumption with lower bond prices the emerging economies are forced to incur in higher levels of debt during periods of economic distress.

between the fundamentals of the economies. The model without financial links predicts all this correlations to be zero.

For example in the data for the period of the crisis the correlation between the spread of Argentina and Argentina's GDP is -0.90. For this same period the correlation between the spread and the output predicted by the contagion model is -0.91 while the model without financial links predicts this correlation to be only -0.20. Regarding the correlation between Argentina's trade balance and Argentina's GDP for the whole period in the data this correlation is -0.59 and both models predicted to be negative and around -0.15. Unfortunately for the period of the crisis the correlations predicted by both models do not exhibit the counter-cyclical behavior observed in the data. The possible explanation for this result consist than the agents in the model are doing as much consumption smoothing as possible during the periods previous to the crisis, while in the data credit markets begin to close in the periods previous to the crisis. With respect to the correlation between Argentina's consumption and Argentina's GDP this correlation is predicted relatively well by both models for the whole period (the correlation is 0.93 in the data, the contagion model predicts this correlation to be 0.97 and the model without financial links predicts it to be 0.90) but the contagion model predicts this correlation much better for the crisis period (in the data this correlation is 0.96, the contagion model predicts it to be 0.99 and the model without financial links predicts it to be 0.68).

Regarding the correlations between the fundamentals of economy A and economy U clearly the contagion model is superior to the model without financial links because as long as the GDPs of the two countries are not correlated (which is the assumption of the model, to focus on the role of the links to explain contagion) such model predicts these correlations to be zero.

For the case of the correlations between the GDP of the country with the crisis (A) and the other country (U) and the spread of the other country, the contagion model predicts the correct sign for the whole period for this correlation but largely underestimate it: in the data the correlation is -0.24 while the model predicts it to be -0.03; however for the period of crisis the contagion model does a good job at explaining this correlation in the sense that is consistent with an important increase in the correlation during the crisis period in relation to the whole period: in the data the correlation is -0.63 while the model predicts it to be -0.80.

A very important correlation is the one between the spreads of the two countries. For the whole period and during the crisis period the contagion model is consistent with the observed positive correlation of these two variables, and with the pattern observed in the

Table 13: EMBI+ Correlations 1994(Q3)-2000(Q4)

	EMBI+ Correlations							
	Argentina	Brazil	Mexico	Morocco	Nigeria	South Africa	V	enezuela
Argentina	1	0.82	0.91	0.96	0.77	0.83	0	.92
Brazil		1	0.78	0.83	0.70	0.49	0	.85
Mexico			1	0.93	0.56	0.83	0	.82
Morocco				1	0.65	0.78	0	.90
Nigeria					1	0.49	0	.77
South Africa						1	0	.79
Venezuela							1	

Source JP Morgan

data of a significant increase in the correlation during the period of crisis, but over-predicts this correlation for the case of the domestic interest rates of Argentina and Uruguay: in the data for the whole period the correlation between the spreads of this rates is 0.18 and the model predicts it to be 0.32, for the period of crisis the correlation is 0.52 in the data and the model predicts it to be 0.88. While this over-prediction for the spreads of the interest rates of Argentina and Uruguay might be seen as a failure of the model it is possible to notice by looking at Table 13 that the correlations predicted by the model are in line with the observe ones for the EMBis+ of pairs of countries like Argentina-Brazil, Argentina-Mexico, Argentina-Morocco, Argentina-Nigeria, Argentina-South Africa, Argentina-Venezuela whose average correlation is for example 0.87 during the period $1994 : Q3 - 2000 : Q4$.

The model is also able to reproduce relatively well the correlations between Argentina's and Uruguay's fundamentals and the wealth of the international investors (which in this context is proxy by the US's GDP). For example, according to the data for the whole period there is a correlation of -0.10 and -0.54 within Argentina's spread and US's GDP during the whole period and the period of the crisis, and the model predicts this correlation to be -0.34 and -0.80 respectively. Also, the correlation between investors' wealth and Argentina's consumption for the whole period and for the period of crisis is 0.60 and 0.72 respectively, and the contagion model predicts these correlation to be 0.31 and 0.84 respectively. As discuss in Lizarazo (2011), the model without contagion links cannot reproduce this behavior.

In general, the results in here suggest that the framework presented in this paper can endogenously account for the transmission of crises across emerging economies. Furthermore, the inclusion of financial links across economies improves the quantitative features of comparable models of endogenous sovereign risk. [23]

6 Conclusions

This paper presents a stochastic dynamic general equilibrium model of default risk that endogenizes the role of external factors in the determination of small open economies' incentives to default, sovereign bond prices, capital flows and default episodes.

The empirical literature on international finance presents evidence that points to a very relevant role for the fundamentals of other emerging countries in the determination of sovereign credit spreads and capital flows to emerging economies. The model in this paper is the first model that endogenously determines sovereign bond prices and at the same time endogenously accounts for contagion of crises.

The endogenization of bond prices and contagion occurs in two ways. First, the consideration of enforcement problems in sovereign debt contracts allows default risk and default incentives to be endogenized; therefore sovereign bond prices can be determined endogenously by the model. Second, the assumption of decreasing absolute risk aversion for investors allows for endogenous financial links across economies that share investors. Together, these two elements build a framework that explains the contagion of crisis. The intuition for contagion is as follows: whenever a negative shock occurs in one country, this shock increases the risk associated with that country which implies an expected future negative wealth shocks for investors. Given decreasing absolute risk aversion, investors' tolerance toward risk decreases following the wealth shock, leading to a portfolio recomposition. Investors shift away from risky investments towards riskless ones.

This explanation for contagion is consistent with the observed behavior of international investors. Investors tend to pull away from other risky countries once one risky country goes into crisis. A testable implication of this explanation would be to test the correlation between the size of investors' losses and the extent of contagion: if the explanation offer in this article is correct, then a larger shock to investors' wealth should induce a more radical portfolio recomposition away from risky investments.

Qualitatively the results of the model are consistent with the empirical evidence of contagion from Argentina to Uruguay: First, sovereign spreads and capital flows to emerging

[23]Comparable models have, as in here, the same dimension for the endowment shocks and the asset position of the economies.

economies are positively correlated across economies. Second, the fundamentals of foreign emerging economies affect the determination of domestic sovereign spreads and capital flows. Third the average financing conditions of an economy are less favorable after other countries have defaulted.

However, the quantitative implementation of the model faces a hurdle in the limitation that the high dimension of the state space imposes on the numerical solution of the model. This problem of the high dimensionality of the state space for the contagion model might be overcome if the steepness of the pricing function can be reduced: even in the contagion model in where bond prices are affected by much more than the own fundamentals of the economy, prices still respond too strongly to changes in the economy's debt level, and therefore it is necessary to have very fine grids for the asset position of the economies in order to capture a great deal of the default action. This need, if satisfied has an explosive effect in the dimensionality of the state space of the model. A logical way to do the reduction in the steepness of the pricing function would be to consider longer term maturities for sovereign bonds modeled as in Arellano and (2008) and Hatchondo and Martinez (2009). Bonds with longer maturities should have a less steep pricing function because investors would care not only about the next period default risk but future period risk as well. Therefore sovereign bond spreads would be smoother since the spreads might be positive even if there is no chance that the economy will default in the next period.

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Appendix

The following proofs assume permanent exclusion of credit markets after a default. Under this assumption the value function of default is independent of γ^L, and W. The quantitative analysis of the model generalizes the results to the case of temporary exclusion. We focus in $b'_j < 0$ (only in this situation is default possible), and the equilibrium of the credit market implies $\theta'_j = -b'_j > 0$. More borrowing implies a more negative b'_j.

Proposition 2 *For any state of the world, S, as the risk aversion of the international investor increases, the emerging economies' incentives to default increase.*

Proof. Considering the case in which the government has not defaulted and assuming an interior solution for the allocation to the emerging economy j's asset the first order condition of the investor's problem is

$$\phi\left(\vartheta'_j\right) = E\left\{-q_j v_c\left(c_L\left(\vartheta'_j\right)\right) + \beta v_c\left(c'_L\left(\vartheta'_j\right)\right) d'_j\right\} = 0.$$

Because the periodic utility of the international investor is of the CRRA type and $\gamma_1^L < \gamma_2^L$, then there exists a concave function $\psi(\cdot)$ such that $v_2\left(c;\gamma_2^L\right) = \psi\left(v_1\left(c;\gamma_2^L\right)\right)$. If $\vartheta'_{j,1}$ is the optimal allocation when $\gamma^L = \gamma_1^L$, and $\vartheta'_{j,2}$ is the optimal allocation when $\gamma^L = \gamma_2^L$ then it holds that

$$\begin{aligned}
\phi_1\left(\vartheta'_{j,1}\right) &= E\left\{-q_j v_{1,c}\left(c_L\left(\vartheta'_{j,1}\right)\right) + \beta v_{1,c}\left(c'_L\left(\vartheta'_{j,1}\right)\right) d'_j\right\} = 0. \\
\phi_2\left(\vartheta'_{j,2}\right) &= E\left\{-q_j v_{2,c}\left(c_L\left(\vartheta'_{j,2}\right)\right) + \beta v_{2,c}\left(c'_L\left(\vartheta'_{j,2}\right)\right) d'_j\right\} = 0.
\end{aligned}$$

Using $v_2\left(c;\gamma_2^L\right) = \psi\left(v_1\left(c;\gamma_2^L\right)\right)$ it is possible to define

$$\phi_2\left(\vartheta'_{j,1}\right) = E\psi'\left[v_1\left(c_L(\vartheta'_{j,1})\right)\right]\left\{-q v_{1,c}\left(c_L\left(\vartheta'_{j,1}\right)\right) + \beta\Upsilon(\vartheta'_{j,1}) v_{1,c}\left(c'_L\left(\vartheta'_{j,1}\right)\right) d'\right\} < 0.$$

where

$$\Upsilon(\vartheta_j') = \frac{\psi'\left[v\left(c_L'(\vartheta_j')\right)\right]}{\psi'\left[v\left(c_L(\vartheta_j')\right)\right]}, \;\; \Upsilon(\vartheta_j') > 0 \;\; \text{and} \;\; \Upsilon'(\vartheta_j') < 0.^{24}$$

The last inequality comes from the fact that both $\Upsilon(\vartheta_j')$ and $\psi'\left(\vartheta_j'\right)$ are positive and decreasing. The inclusion of these functions in the previous equation implies that $\phi_2\left(\vartheta_{j,1}'\right)$ is lower than $\phi_2\left(\vartheta_{j,2}'\right)$ because $\Upsilon'(\vartheta_j')$ and $\psi'\left(\vartheta_j'\right)$ give little weight to the realizations of $d_j{}' = 1$, and high weight to the realizations of $d_j{}' = 0$. Therefore $\phi_2\left(\theta_{j,2}'\right) > \phi_2\left(\theta_{j,1}'\right)$.

The concavity of $V^L(\cdot)$ implies that given q_j and the risk of default δ_j, $\phi\left(\theta_j'\right)$ is a decreasing function and as a consequence $\theta_{j,2}' < \theta_{j,1}'$ which in equilibrium implies $b_{j,2}' > b_{j,1}'$.

Then for any state of the world S, taking as given q_j and (δ_j), a higher γ^L would result in the investor allocating a lower proportion of her portfolio to each of the economies' sovereign bonds. Therefore, when the investor is less risk averse there are financial contracts that are available to each emerging economy which are not available when the investor is more risk averse. Consequently given q_j and δ_j then $V_{j,1}^C\left(S;\gamma_1^L\right) \geq V_{j,2}^C\left(S;\gamma_2^L\right)$

Because the utility of autarky for the emerging economy does not depend on γ^L, it is clear that if for some S, default is optimal for economy j when $\gamma^L = \gamma_1^L$, then for the same S default would be optimal when $\gamma^L = \gamma_2^L$. Additionally, because incentives to default would be higher whenever $\gamma^L = \gamma_2^L$, than when $\gamma^L = \gamma_1^L$ at equilibrium $\delta_j\left(S, b_j{}'; \gamma_2^L\right) > \delta_j\left(S, b_j{}'; \gamma_1^L\right)$, and therefore $q_j\left(S, b_j{}'; \gamma_2^L\right) < q_j\left(S, b_j{}'; \gamma_1^L\right)$. ■

Proposition 3 *Default sets are shrinking in the assets of the representative investor. For all $W_1 < W_2$, if default is optimal for b_j in some states y_j, given W_2 then default will be optimal for b_j for the same states y_j, given W_1 therefore $D_j\left(b_j \mid W_2, \psi, \{s_k\}_{k=1,k\neq j}^J\right) \subseteq D_j\left(b_j \mid W_1, \psi, \{s_k\}_{k=1,k\neq j}^J\right)$*

Proof. Because the periodic utility of the international investor exhibit DARA $v\left(W_1, \vartheta_j'\right)$ is a concave transformation of $v\left(W_2, \vartheta_j'\right)$ so if $\vartheta_{j,1}'$ is the optimal allocation

[24] Taking as given the portfolio allocations of the investor to other emerging economies the derivative of $\Upsilon(\vartheta_j')$ is given by

$$\Upsilon'(\vartheta_j') = \frac{\psi''[v(c_L'(\vartheta_j'))]v_c(c_L'(\vartheta_j'))\frac{\partial c_L'(\vartheta_j')}{\partial \vartheta_j'} \; - \; \psi''[v(c_L(\vartheta_j'))]v_c(c_L(\vartheta_j'))\frac{\partial c_L(\vartheta_j')}{\partial \vartheta_j'}\Upsilon(\vartheta_j')}{\psi'\left[v\left(c_L(\vartheta_j')\right)\right]} < 0.$$

when $W = W_1$, and $\vartheta'_{j,2}$ is the optimal allocation when $W = W_2$, it is possible to define $v_1\left(\vartheta'_{j,1}\right) = v\left(W_1, \vartheta'_{j,1}\right)$ and $v_2\left(\vartheta'_{j,2}\right) = v\left(W_2, \vartheta'_{j,2}\right)$, where $v_1\left(\vartheta'_j\right) = \psi\left(v_2\left(\vartheta'_j\right)\right)$. The first order conditions of the investor are

$$\begin{aligned}
\phi_1\left(\vartheta'_{j,1}\right) &= E\left\{-q_j v_{1,c}\left(c_L\left(\vartheta'_{j,1}\right)\right) + \beta v_{1,c}\left(c'_L\left(\vartheta'_{j,1}\right)\right) d_j{}'\right\} = 0, \\
\phi_2\left(\vartheta'_{j,2}\right) &= E\left\{-q_j v_{2,c}\left(c_L\left(\vartheta'_{j,2}\right)\right) + \beta v_{2,c}\left(c'_L\left(\vartheta'_{j,2}\right)\right) d_j{}'\right\} = 0,
\end{aligned}$$

and therefore

$$\phi_1\left(\vartheta'_{j,2}\right) = E\psi'\left[v_2\left(\vartheta'_{j,2}\right)\right]\left\{-q_j v_{2,c}\left(c_L\left(\vartheta'_{j,2}\right)\right) + \beta\Upsilon(\vartheta'_{j,2}) v_{2,c}\left(c'_L\left(\vartheta'_{j,2}\right)\right) d_j{}'\right\} < 0.$$

$\Upsilon(\vartheta')$ is defined as before, and as before the inequality comes from the fact that $\Upsilon(\vartheta'_j)$ and $\psi'(\vartheta'_j)$ are both positive and decreasing. Therefore $\phi_1\left(\vartheta'_{j,2}\right) < \phi_1\left(\vartheta'_{j,1}\right)$.

Again the concavity of $V^L(\cdot)$ implies that given q_j and δ_j, $\phi\left(\vartheta'_j\right)$ is a decreasing function, and as consequence $\vartheta'_{j,2} > \vartheta'_{j,1}$ which in equilibrium implies $b'_{j,2} < b'_{j,1}$.

Then for any S and taking as given q_j and δ_j, a lower level of W would result in this agent allocating a lower proportion of her portfolio to the emerging economies' sovereign bonds. Therefore, when the investor is more wealthy there are financial contracts that are available to the emerging economy that are not available when the investor is less wealthy. Consequently, given q_j and δ_j then $V^C_{j,1}(S_{-W}; W_2) \geq V^C_{j,2}(S_{-W}; W_1)$. [25]

Because the utility of autarky for the emerging economy does not depend on W, it is clear that if for some b_j in some states y_j, default is optimal when $W = W_2$, then for the same states y_j default would be optimal when $W = W_1$. Because incentives to default would be higher whenever $W = W_1$, than when $W = W_2$ at equilibrium $\delta_j\left(S, b_j{}'; W'_1\right) > \delta\left(S, b_j{}'; W'_2\right)$, and therefore $q\left(s, b_j{}'; W'_1\right) < q\left(s, b_j{}'; W'_2\right)$. ■

Proposition 6 *There is a wealth channel of contagion. Proposition 2 implies that if economy k which is in the investor's portfolio defaults in her debts, incentives to default for economy j which is also in the investor's portfolio increase.*

Proof. If economy k defaults in her debts with the investor, the wealth of this agent will be $(W \mid d_k = 0) = \theta^{TB'} + \sum_{m=1, m\neq k}^{J} \theta'_m$, which is lower than the wealth for economy k if

[25] S_{-W} corresponds to all the state variables of the world except for the wealth level of the investors.

she decides not to default, which is $(W \mid d_k = 1) = \theta^{TB'} + \sum\limits_{m=1, m\neq k}^{J} \theta'_m + \theta'_k$. Therefore

$$V_j^C\Big(y_j, b_j, \{y_m\}_{m=1}^J, \{b_m\}_{m=1}^J, (W \mid d_k{=}1)\Big) > V_j^C\Big(y_j, b_j, \{y_m\}_{m=1}^J, \{b_m\}_{m=1}^J, (W \mid d_k{=}0)\Big)$$

which implies that emerging economy $j's$ incentives to default are larger when some economy k which shares investors defaults. ■